PSY 201: Statistics in Psychology

Lecture 12
Probability
Why casinos make money.

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DESCRIPTIVE STATISTICS

- most of what we have discussed so far is called descriptive statistics
  - distributions
  - graphs
  - central tendency
  - variation
  - correlation
- **describe** sets of data

INFERENTIAL STATISTICS

- given a set of data from a sample
- we want to **infer** something about the entire population
  - mean
  - standard deviation
  - correlation
  - ...
- never with certainty, but with **probability**

PROBABILITY

- number between 0 and 1
- probability of event A is written as

$$P(A)$$

- if

$$P(A) = 1.0$$

- it indicates with certainty that event A will happen
- if

$$P(A) = 0$$

- it indicates with certainty that event A will **not** happen

PROBABILITY LAWS

- there are specific rules to probability
- we want to know the probability of many events, pairs of events, contingent events,...
- how to calculate probabilities depends upon
  - Complements
  - Mutually exclusive compound events
  - Nonmutually exclusive events
  - Statistically independent joint events
  - Statistically dependent joint events

SINGLE EVENTS

- precise definition requires high-level mathematics
- intuitive definition is that probability of a single event is the ratio of the number of possible outcomes that include the event to the total number of possible outcomes

$$P(\text{a die coming up 3}) = \frac{\text{Number of outcomes that include 3}}{\text{Total number of outcomes}}$$

$$P(\text{a die coming up 3}) = \frac{1}{6} \approx 0.167$$

1 2 3 4 5 6

COMPLEMENTS

- suppose we know the probability $P(A)$, where A is some event
- then if $\overline{A}$ represents "not A" (called the complement of A)

$$P(\overline{A}) = 1.0 - P(A)$$

- when $A =$ turning up a 3 on a die, $\overline{A}$ means turning up anything other than a 3
- since $P(A) = 0.167$
- $P(\overline{A}) = 1.0 - 0.167 = 0.833$

1 2 3 4 5 6

COMPOUND EVENTS

- sometimes we know the probability of two events A and B, and we want to know the probability of event A or B
- e.g.

$$P(\text{turning up a 3 or a 4 on a die})$$

- these are **mutually exclusive events**
- one or the other

MUTUALLY EXCLUSIVE

- for mutually exclusive compound events, calculating the probability of the compound is easy
- consider probability of rolling numbers on a die

$$P(\text{a 3 or a 4}) = P(3) + P(4)$$

$$P(\text{turning up a 3 or a 4 on a die}) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

1 2 3 4 5 6

- in general, if A and B are mutually exclusive

$$P(A \text{ or } B) = P(A) + P(B)$$

NONMUTUALLY EXCLUSIVE

- sometimes events are **not** mutually exclusive
- e.g.
  - A = turning up a number ≤ 3 on a die: $P(A) = \frac{1}{2}$
  - B = turning up an odd number on a die: $P(B) = \frac{1}{2}$
- what is $P(A \text{ or } B)$?

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

- cannot just add probabilities because numbers common to A and B get counted twice!

NONMUTUALLY EXCLUSIVE

- subtract out common probability

$$P(\text{number} \leq 3 \text{ or odd}) =$$

$$P(\leq 3) + P(\text{odd}) - P(\leq 3 \textbf{ and } \text{odd}) =$$

$$\frac{1}{2} + \frac{1}{2} - \frac{2}{6} = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$$

- in general

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

- when the events are mutually exclusive, $P(A \text{ and } B) = 0$, and we get the rule for mutually exclusive events

JOINT EVENTS

- if we know $P(A)$ and $P(B)$, what is $P(A \text{ and } B)$?
- both events must occur (simultaneously or successively)
- e.g.

$$P(3 \text{ on a die and HEAD on a coin flip})$$

STATISTICAL INDEPENDENCE

- events are independent if the occurrence of one event does not affect the probability of the other event occurring
- e.g., rolling a 3 on a die has no effect on whether or not a coin will come up HEADS

$$P(\text{3 on die}) = \frac{1}{6}$$

$$P(\text{HEADS}) = \frac{1}{2}$$

- so

$$P(\text{3 and HEADS}) = P(3) \times P(\text{HEADS})$$

$$P(\text{3 and HEADS}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

MULTIPLICATION

- why multiply probabilities of joint events?
- probability is ratio of the number of outcomes including an event to the total number of possible outcomes
- for the joint event "3 on a die and HEADS", the possible outcomes are

1H, 2H, 3H, 4H, 5H, 6H
1T, 2T, 3T, 4T, 5T, 6T

- count up the possibilities!

SAMPLING WITH REPLACEMENT

- suppose we have 10 numbered balls in a jar
- the probability of drawing ball 3 is $\frac{1}{10}$
- if we put the ball back, the probability of drawing ball 3 again is $\frac{1}{10}$ (same for any ball)
- each event (drawing ball 3) is independent from previous events
- in general for independent events A and B,

$$P(A \text{ and } B) = P(A) \times P(B)$$

SAMPLING WITHOUT REPLACEMENT

- many times the probability of an event **does** depend on other events
- e.g., suppose we have ten numbered balls in a jar
- the probability of drawing ball 3 is $\frac{1}{10}$
- suppose we draw ball 2; leaving nine balls in the jar
- the probability of drawing ball 3 is now $\frac{1}{9}$

CONDITIONAL PROBABILITIES

- we can describe the effect of other events by identifying conditional probabilities
- e.g.

$$P(\text{drawing ball 3 given that ball 2 was already drawn})$$

$$P(\text{ball } 3|\text{ball } 2)$$

- in general the probability of event A, given event B is written as

$$P(A|B)$$

- no direct way of calculating from $P(A)$ or $P(B)$

NONINDEPENDENT EVENTS

- when
$$P(A) = P(A|B)$$
- we say events A and B are independent
- otherwise the events are nonindependent (dependent)

JOINT PROBABILITY

- if we know $P(A)$ and $P(B|A)$ then we can calculate the joint probability

$$P(A \text{ and } B) = P(A)P(B|A)$$

- if we know $P(B)$ and $P(A|B)$ then we can calculate the joint probability

$$P(A \text{ and } B) = P(B)P(A|B)$$

- same number!
- if events are independent, this rule is the same as before because

$$P(A|B) = P(A)$$

EXAMPLE

- what is the probability of drawing ball 2 and then ball 3 from a jar with ten numbered balls?
- we know that

$$P(\text{drawing ball 2 from the full jar}) = \frac{1}{10}$$

$$P(\text{drawing ball 3}|\text{ball 2 is drawn from the full jar}) = \frac{1}{9}$$

- so

$$P(\text{drawing ball 3 and drawing ball 2}) =$$

$$P(\text{drawing ball 2 from the full jar})\times$$

$$P(\text{drawing ball 3}|\text{ball 2 is drawn from the full jar}) =$$

$$\frac{1}{10} \times \frac{1}{9} = \frac{1}{90}$$

RANDOMNESS

- we assume coin flips, rolling dice, samples from jars are **random** events
- unpredictable for a specific instance
- predictable on average over lots of samples (likelihood of happening)
- randomness is sometimes a good thing

CONCLUSIONS

- probability
- mutually exclusive events
- compound events
- independence

NEXT TIME

- review for exam
- **SECTION EXAM 1**
- fun problems with probability