Homework 4
Due: March 18, 2024

The power law of practice (Snoddy, 1928) states that reaction time for a task decreases according to a *power function*:

$$RT = aP^{-b} + c$$

Where RT is response time and P is the trial number (starting at 1). The parameter c is the asymptotic RT value and the parameter a is set so that $a+c$ is the RT for the first trial. The parameter b is the exponent that sets the shape of the curve. The black line in the plot below shows a power function for a=100, b=2, and c=500.
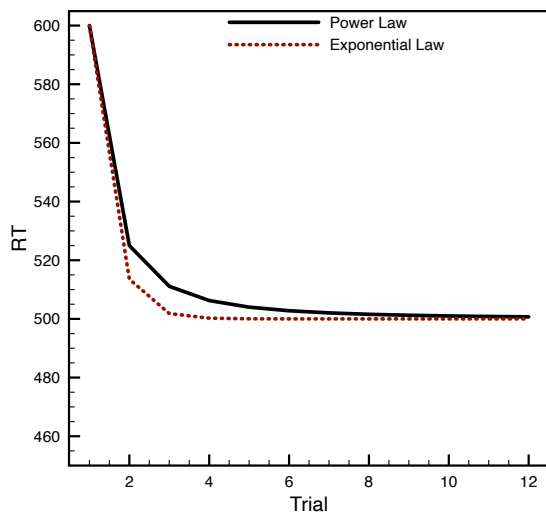
An alternative to the power law is an exponential law, which has the following equation:

$$RT = ae^{-b(P-1)} + c$$

The parameters a and c have a similar interpretation as for the power law, and the exponent b again sets the shape of the curve over the specified region. The red dotted line in the plot shows an exponential function with a=100, b=2, and c=500. A brief discussion of these models is at

https://en.wikipedia.org/wiki/Power_law_of_practice

For this homework, you will run Bayesian analyses to compare the two laws against a "no-law" model. The data come from an "Implicit Learning" experiment, which you can experience at

https://coglab.cengage.com/

(You will be prompted to log in and can use ID: GregTest-99, Password: 12345678, then navigate to LABS->Implicit learning.) Participants are randomly assigned to either a random or

pattern condition. We care about the pattern condition, where implicit learning can occur. The task on each trial is to respond as quickly as possible to identify the location of a red circle that appears on the screen. The trials are split into 6 blocks, and a particular pattern repeats within each block. A common finding is that people implicitly learn the pattern (as evidenced by faster response times across blocks), even though they are unaware of such learning.

From the class website, get the CSV file named "Learning.csv". For each of the 87 participants it reports the mean response time (averaged across the 24 trials) for each block.

Use the *ulam( )* function in the *rethinking* package to build and compare four models:

1. A "power law" model that supposes RT follows the power law equation, above, as a function of block. Ignore differences across participants.
2. An "exponential" model that supposes RT follows the exponential equation, above, as a function of block. Ignore differences across participants.
3. A "no law" model that simply estimates the RT for each block. Ignore differences across participants.

   To complete the assignment, submit R source code that generates all these models and provides answers to the following questions. In addition, you should submit a document (MS Word, or PDF) that directly answers the questions.

1. Check the r-hat values for each model parameter. If r-hat is bigger than 1.05, change the *ulam( )* settings and run it again. [Hint: if necessary, increase "iter" and "warmup". You can get a general idea of priors that might work for *a* and *c* by looking at the data. For parameter *b* you probably want a prior that has most of its distribution between 0 and 10.]
2. After fitting the power law or exponential law models Stan might indicate something like the following:
   Warning: 115 of 12000 (1.0%) transitions ended with a divergence.
   See https://mc-stan.org/misc/warnings for details.
   The information there (following a few links) suggests that divergence is a pretty serious problem and that you might want to increase a control parameter you can send to STAN called *adapt_delta,* which cannot be larger than 1. In your *ulam* call, try adding: control=(list(adapt_delta=0.999)). The divergence warning might also indicate that your priors are too broad. Adjust your code so that it runs without the divergence warning. Explain what you did.
3. In a direct comparison of the model WAIC values, which model best predicts future data sets? How much confidence do you have in this evaluation?
4. For each model, use the link function to generate predicted mean RT for the 6 blocks. Make a plot of the data means as points for each block and superimpose lines for the different models.
5. Use the link function to generate the predicted mean RT for a hypothetical 7th block using the power law and exponential law models. Use the WAIC weights to make a model average prediction.
6. Write up a "Results" section as if it were part of a journal article.