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The Function of Visual Search and Memory in Sequential Looking Tasks

JULIE EPELBOIM,*† ROBERT M. STEINMAN,* EILEEN KOWLER,‡ MARK EDWARDS,* ZYGMUNT PIZLO,§ CASPER J. ERKELENS,¶ HAN COLLEWIJN‖

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Eye and head movements were recorded as unrestrained subjects tapped or only looked at nearby targets. Scanning patterns were the same in both tasks: subjects looked at each target before tapping it; visual search had similar speeds and gaze-shift accuracies. Looking, however, took longer and, unlike tapping, benefitted little from practice. Looking speeded up more than tapping when memory load was reduced: memory was more efficient during tapping. *Conclusion*: eye movements made when only looking are different from those made when tapping. Visual search functions as a separate process, incorporated into both tasks: it can be used to improve performance when memory load is heavy.

Saccades Visual search Memory Visuomotor coordination

INTRODUCTION

Attempts have been made to use eye movements as indicators of mental operations for more than a century (Javal, 1878). Such attempts were often unsuccessful. They shed little light on cognitive processes, in part because eye movement patterns vary greatly among individuals, tasks and even within the same individual and task. Difficulties, however, go beyond issues of variability. In order to infer mental operations from eye movements, it has been necessary to make a number of questionable assumptions.

One of these assumptions is that locus of fixation indicates either: (i) the place in the visual field that contains the information that is being processed mentally, or (ii) the particular type of mental operation taking place (e.g. comparing object colors vs object shapes in a matching task). Another assumption is that the amount of time spent looking at a particular locus is proportional to the amount of mental processing of information at that locus. These assumptions, although frequently made, are difficult to verify because mental processing is private and cannot be measured directly [see Viviani (1990) and Suppes (1990) for a critique of these and other difficulties encountered when eye movements are used to infer mental processing].

Despite the fact that prior attempts to determine moment-by-moment mental operations by looking at individual eye movements have been problematic, examining global patterns of eye movements have yielded some interesting and useful insights into the mental world of subjects in visuomotor experiments. Ballard, Hayhoe and Pelz (1995) and Hayhoe, Ballard and Whitehead (1993), for example, found that when subjects were asked to copy meaningless shapes made by arranging an assembly of colored blocks, they looked at the shape to be copied an average of twice for each block they arranged, at least while they were arranging the first three blocks. This strategy was pervasive and suggested that, at least in this type of task, the subjects were using the visible display as an extension of their memory. Rather than memorizing the locations of several blocks at a time, the subjects chose to look at the model repeatedly while the copy was built. This strategy changed when eye movements were made more expensive by placing the model farther away from the work area within which the copy was being built. In this case subjects did not look at the model as often, and used their memory more.

Another example of using global eye movement patterns to study mental processes is the work on arithmetic by Suppes, Cohen, Laddaga and Floyd (1983). Suppes *et al.* (1983) classified the eye movements that were made while subjects solved arithmetic problems into classes corresponding to specific operations, e.g. "go to the next number", "go back by one", "skip the next number", etc. They found that the frequencies of different operations varied with the difficulty of the problem and

*Department of Psychology, University of Maryland at College Park College Park, MD 20742-4411, U.S.A.
†To whom all correspondence should be addressed [*Email* yulya@brissun.umd.edu].
‡Department of Psychology, Rutgers University, New Brunswick, NJ 08903, U.S.A.
§Department of Psychological Sciences, Purdue University, West Lafayette, IN 47907, U.S.A.
¶Department of Biophysics, University of Utrecht, Utrecht, The Netherlands
‖Department of Physiology I, Erasmus University, Rotterdam, The Netherlands

with the mathematical ability of the subject. In particular, they found that with more difficult problems and less skilled subjects, there was a higher probability of eye movements that did not fit in to any of the classes described above. Instead, the subjects performed seemingly unproductive operations such as skipping more than two numbers, or looking at blank areas on the page. Suppes *et al.* (1983) concluded that these out-of-sequence operations represented "wandering" of the eye that occurred while the subject was not sure of what to do next—he wandered as he wondered. Epelboim, Booth and Steinman (1994a) showed that when a similar classification was used to classify eye movements made while reading a text, the out-of-sequence operations, such as going back to the previous sentence, or skipping forward by more than two words, represented a different of type mental process than such operations seemed to represent when arithmetic problems were solved. Specifically, Epelboim *et al.* (1994a) found that the probability of out-of-sequence fixations *decreased* with slower readers and more difficult reading tasks, opposite to what had been observed during arithmetic. Subjects looked away from the numbers when the arithmetic became more difficult, whereas they seemed to fear to stray from the straight and narrow path provided by the text when reading became more difficult. This allowed Epelboim *et al.* (1994a) to conclude, on the basis of their comparison of global eye movement characteristics, that reading and doing arithmetic are fundamentally different mental operations. This conclusion is hardly counter-intuitive, but it is one of only a very few examples where eye movement patterns were used successfully to show quantitatively what had previously been suspected only intuitively. A less obvious finding of the Epelboim *et al.* (1994a) study, also obtained by comparing eye movement patterns, was that reading normal text and reading text with the spaces between words removed are not fundamentally different. This finding was unexpected because recent theories of reading eye movements place great importance on spaces between words as guides for eye movements during reading (e.g. O'Regan, 1990; Rayner, 1993).

In the experiments described in the present paper, we continued the tradition of trying to infer mental operations from eye movement patterns and used eye movement characteristics to compare another pair of tasks. In one task, free-headed subjects tapped sequences of targets located in front of them. In the second task, they only looked at sequences of targets without doing anything else to them. We wanted to know whether adding hand movements changed the pattern of eye movements or if during tapping, the subjects simply performed two independent tasks in parallel—looking at the targets and tapping them. We had some reason to expect that adding arm movements to a looking task would alter eye movement patterns and improve performance. This expectation was based on a series of studies in which subjects performed relatively natural visuomotor tasks with their heads and torsos unrestrained in visually rich, well-lighted environments with targets located within the arm's reach. As conditions and tasks became more natural, eye movements became faster and more effective (e.g. Kowler, Pizlo, Zhu, Erkelens, Steinman & Collewijn, 1992; Collewijn, Steinman, Erkelens, Kowler & Van der Steen, 1992a; Collewijn, Steinman, Erkelens, Pizlo & Van der Steen, 1992b; Erkelens, Van der Steen, Steinman & Collewijn, 1989; also see Steinman, Kowler & Collewijn, 1990, for a discussion of the importance of using natural environments and tasks in oculomotor research).

Here, we report that there are both similarities and differences in the global eye movement patterns used in both looking and tapping tasks. Spatial scanning patterns were the same regardless of whether the subjects only looked at the targets or looked at them to guide their tapping arm movements. Also, both tasks included out-of-sequence eye movements that were used to search for the next target. In both tasks, these out-of-sequence of eye movements had the same timing characteristics, and showed the same decrease in the frequency of occurrence with practice. Subjects looked at each target in sequence in both tasks, but they were not aware that they were using this strategy in the tapping task, insisting that targets were skipped once the pattern became familiar. Despite these similarities they did not perform the two tasks at the same speed. Completing tapping sequences took much less time than completing looking-only sequences of the same length. Furthermore, tapping a sequence of targets got faster with practice, whereas simply looking at a sequence of targets did not. These results make it possible to conclude that looking at targets to serve some purpose (tapping) is fundamentally different from looking at targets for their own sake. We also conclude that visual search acts as a separate, but not independent, process that is incorporated into these two quite different tasks in the same way.

METHOD

Subjects

Four subjects (CE, HC, RS, and ZP) participated. All were experienced eye-movements subjects, but did not have any prior experience whatsoever with the Worktable or the tapping and looking tasks. The data reported here include their very first attempts at these tasks.

Data Collection

Apparatus

The Maryland revolving field monitor (MRFM) apparatus consists of three subsystems [Fig. 1(a)].

Subsystem 1. The revolving field monitor/sensor coil subsystem (RFM) is used to record angular positions of the eyes and the head. This system consists of two major parts: (i) a machine that produces three, mutually perpendicular, magnetic fields that revolve at different frequencies (976, 1952, and 3904 Hz) inside the MRFM chamber; (ii) sensor-coil devices which, when placed inside the chamber, carry an induced current that is

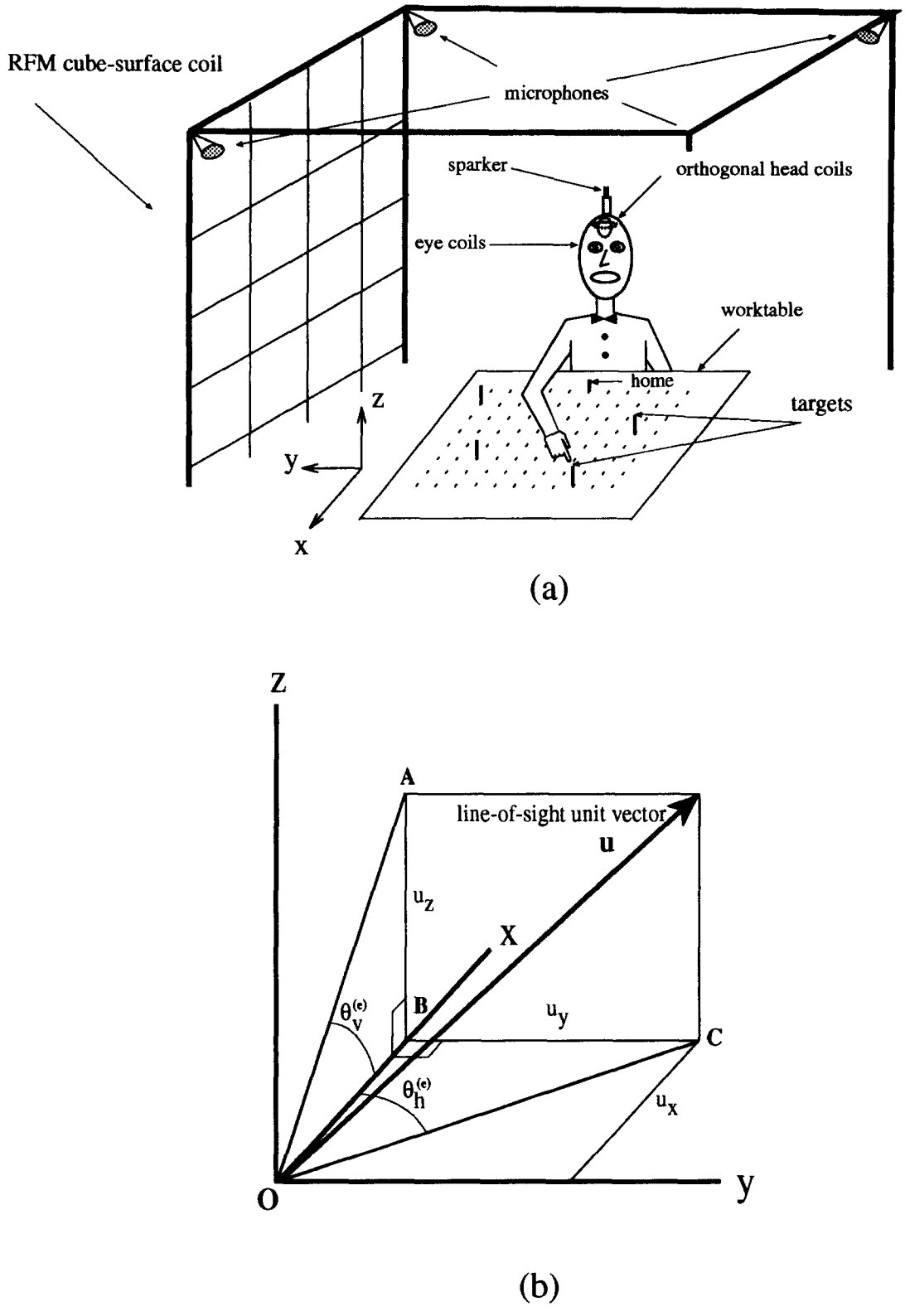


FIGURE 1. (a) The Maryland revolving-field monitor apparatus. (b) Determining the line-of-sight unit vector in Worktable coordinates. See text for an explanation.

dependent on the spatial orientation of the coil. A bank of electronics reads and processes this induced current and also controls the magnetic fields.

Each revolving field is produced by two sets of five-element, a.c.-current-carrying coils mounted on a cubical frame 2.5 m on a side in a "cube-surface coil" arrangement (Rubens, 1945). The magnetic field produced by this arrangement is spatially homogeneous throughout a large fraction of the volume inside the cubical frame. When a sensor coil is placed inside the RFM chamber, a.c. current is induced in the coil by the revolving magnetic fields. The total magnetic field generated by the RFM is the superposition of the three revolving fields. The total a.c. current induced in each

sensor coil immersed in this field is a superposition of three sinusoids, each having a different frequency and amplitude. The electronics record this current, separate out the components of different frequencies, and compute the difference between the phase of a particular component and that of the corresponding component of induced a.c. current in the appropriate reference coil. The set of three phase differences, one for each revolving field, determine the absolute orientation of the sensor coil. This is the case because each phase difference is equal to the angle between the orthographic projections of the unit normal vectors of the sensor coil and the appropriate reference coil onto the appropriate revolving-field plane. The phase detection method was introduced by Collewijn (1977). The precision of angle measurement of the RFM is better than 1 min arc with linearity better than 0.01%. Sampling rate was set at 488 Hz, yielding an effective bandwidth of 244 Hz.

Sensor coils embedded in a silicone annulus (Skalar–Delft) were attached to each eye by suction and used to measure horizontal and vertical eye rotations. A head coil apparatus, which consisted of two approximately perpendicular sensor coils, was used to measure three-dimensional rotations of the head. The larger coil, approximately parallel to the head's frontoparallel plane, was used to measure horizontal and vertical head rotations (yaw and pitch), and the smaller coil, approximately parallel to the head's sagittal plane, was used to measure torsional (roll) rotations of the head (see Fig. A1 in the Appendix).

Subsystem 2. The sparker tracking system (STS) is used to track three-dimensional translations of the head. It contains two major components: (i) a set of four microphones mounted on a rectangular frame situated above the cubical frame of the RFM; (ii) a device called a "sparker", mounted on the subject's head. A sparker is a rod with electrodes at one end separated by a tiny air gap. A large potential difference is placed across these electrodes at the rate of 61 Hz, causing a spark to jump across each time (sparker strobe). At each sparker strobe, a sharp, high-pitched sound (60 kHz on the leading edge) is emitted, and then detected by the microphones. The electronic circuitry controlling the sparker and the microphones computes the delay between the time the voltage was applied to the sparker and the time the sound was detected by each microphone. These delays are converted to distances using the principle that the distance the sound travels is proportional to the time-delay introduced by the speed of sound in air. The precision of the distance measurement is <0.2 mm and its accuracy is about 1 mm.

Subsystem 3. The Worktable serves to provide stimuli for the experiments. It is made of plastic and has a flat surface containing a grid of 154 wells arranged in 11 rows and 14 columns. The rows and columns of wells are 45 mm apart. Microswitches are located at the bottom of each well. Rods topped with LEDs of different colors served as targets that could be placed in the wells on the Worktable. When a subject tapped one of the targets, a microswitch was activated and the time of the tap was recorded to the nearest 2 msec.

A target, without an LED, placed in a well near to the subject was designated "home" [see Fig. 1(a)]. The top of the home target serves as the origin of the Worktable-coordinate-system. The definitions of the axes of this coordinate system are depicted in Fig. 1(a).

The MRFM collected and stored data from its three subsystems in discrete "bursts", each containing 12 signals produced by the RFM, STS and the Worktable. 488 RFM bursts were obtained and stored each second. Every eighth RFM burst contained new sparker values because there were only 61 sparker strobes/sec.

Calibrations

Three types of calibrations were performed.

Sparker calibration. The STS had to be calibrated to find a conversion matrix between distance outputs of the microphones and Worktable coordinates. This calibration needed to be performed only once because the position of the Worktable and the sparker microphones remained constant throughout the series of experiments described in this paper. The conversion matrix was determined by placing sparkers of two different heights in 18 different locations on the Worktable. The conversion matrix between sparker outputs and their known Worktable coordinates was determined using a standard least-squares estimation technique. Once it was determined, this matrix could be used to convert the STS outputs of a sparker placed in an arbitrary location within the MRFM chamber into Worktable coordinates, with accuracy of better than 1 mm.

Finding each subject's sighting centers with the head in a known, on-biteboard location. In order to calculate where the subject was looking with respect to targets on the Worktable, it was necessary to find the Worktable coordinates of the sighting center of each eye when his head was in a reproducible, known position on a biteboard. The concept of a point within the eye that is fixed in relation to the head and that lies along the line-of-sight is founded in previous work (Park & Park, 1933) at least for horizontal movements of the eye. In Park and Park, this point was found to be approx. 13.5 mm behind the front surface of the cornea along the line-of-sight. In order to locate this point for each subject and each eye, a biteboard holder was attached to the Worktable and a thin tube with a sighting pinhole at one end and a tiny LED at the other end was mounted on a movable platform on the Worktable in front of the subject. With the head on the biteboard, the subject used one eye to look through the pinhole in the tube at the LED located at the other end. The sighting-tube platform was moved until the LED could be seen clearly through the sighting pinhole. The sighting tube was positioned on the Worktable such that when the subject saw the LED, his line-of-sight was coincident with the axis of the sighting tube, and parallel to the Worktable x-axis. Then the subject closed his eye and the tube was moved towards him (along the line-of-sight) until it touched his eyelid. The subject then got off the biteboard and a sparker was

positioned 14 mm (13.5 mm behind the cornea + 0.5 mm allowed for the thickness of the eyelid) in front of the end of the tube nearest to the subject. Sparker data were collected and converted to Worktable coordinates, producing an estimation of the location of one eye's sighting center with the head in a reproducible position on the biteboard. The procedure was repeated for each subject and each eye three times and the median sparker values for each microphone were used to compute sighting center coordinates that were then used in all subsequent calculations.

Finding offsets of the eye and head coils. At the beginning of each recording session, eye and head coil offset calibration measurements were performed. This was necessary because the placements of the annuli on the eyes and the head coil apparatus on the head varied from session to session. Generally, these measurements were performed during the first two trials of a session—one trial for each eye. These calibration trials were called "mirror trials".

The same biteboard holder, used to find the sighting centers, was used in the mirror trials. A front-surface mirror was attached to the biteboard holder in front of the subject and perpendicular to the Worktable x-axis. During each mirror trial, the subject occluded one eye and looked with the other eye at the reflection of his pupil in the mirror. This caused the line-of-sight of the seeing eye to be parallel with the Worktable x-axis. The procedure was repeated at least once for each eye during each recording session. The mean values for horizontal and vertical eye-angles recorded during the mirror trials were used as zero-angle reference values and subtracted from all subsequent eye data.

The mean sparker position during the mirror trials was used to find vectors from the positions of the sighting centers with the head on the biteboard to the mirror-trial position of the sparker.

Finding the Worktable coordinates of line-of-sight vectors at an arbitrary RFM burst

Definition of the subject's "line-of-sight". There are several ways of defining the line-of-sight of a subject (Alpern, 1962). Most of these definitions require knowledge of the foveal location—a quantity that was not measured in our experiments. In our study, the line-of-sight was defined as the line that was parallel to the Worktable x-axis and that passed through the on-biteboard sighting-center position (determined previously as described above) while the subject fixated his pupil during the mirror trial of a session. This operational definition of the line-of-sight is based on the assumption that the line-of-sight is fixed relative to the subject's eye. Thus, once the line-of-sight is known (defined) for a given orientation and spatial location of the eye (henceforth, the eye's configuration), the line-of-sight can be found for any arbitrary configuration, as long as all translations and rotations that moved the eye from the known line-of-sight configuration to the arbitrary configuration can be determined.

Another fundamental assumption was that the subject's head, head coils, and head sparker moved as a rigid body throughout a given session. Given this assumption, it was possible to determine the line-of-sight of an eye at an arbitrary burst of the RFM if the readings of the eye coil, head coils, and sparker were known both during the mirror trial and at the arbitrary RFM burst and if, furthermore, the sighting-center position of that eye was known during the mirror trials. All of these quantities were measured during our experiments.

Determination of the lines-of-sight of the two eyes at an arbitrary RFM burst from the lines-of-sight during the mirror trials consisted of determining: (i) the new location of the sighting centers at the arbitrary RFM burst relative to their on-biteboard locations; (ii) horizontal and vertical eye angles at the arbitrary RFM burst relative to the horizontal and vertical eye angles during the mirror trials. Since the sighting-center locations of both eyes during the mirror trials were known relative to the space-fixed Worktable coordinate system, the locations of the sighting centers at the later time were also determined relative to this coordinate system. The lines-of-sight at the later time were determined by finding the lines that passed through the new sighting-center locations and which were parallel to the on-biteboard lines-of-sight rotated in the same fashion as was required to bring the eyes to their new orientations.

Finding the new sighting-center locations. The new sighting-center locations can be found relative to their on-biteboard locations by noting that the head undergoes both a translation and a rotation. The translation can be determined from the on-biteboard sparker readings and those measured at the arbitrary RFM burst. The vector from the sparker tip to the sighting center of a given eye, $\mathbf{x}_{eb}$, in the on-biteboard case can be written as

$$\mathbf{x}_{eb} = \mathbf{r}_{eb} - \mathbf{r}_{hb}, \tag{1}$$

where $\mathbf{r}_{eb}$ is the on-biteboard location of the sighting center and $\mathbf{r}_{hb}$ is the on-biteboard location of the sparker tip.

Now consider a situation in which the subject's head is oriented such that the large head coil is perfectly aligned with the yz-plane of the Worktable coordinate system, the line formed by the intersection of the two head coils is exactly parallel with the Worktable z-axis, and the sparker tip has the same location as in the on-biteboard case. The large head coil readings in this case will be zero on both the horizontal and vertical meridians and the torsional angle of the smaller coil will also be zero. This configuration will be referred-to hereafter as the "standard configuration". Any arbitrary sparker-tip–sighting-center vector differs from its standard-configuration counterpart (denoted by $\mathbf{x}_{e0}$) by a pure rotation which depends only on the head coil readings at the arbitrary burst. This can be expressed as follows

$$\mathbf{x}_e = R(\theta_h, \theta_v, \theta_t)\mathbf{x}_{e0}, \tag{2}$$

where $\mathbf{x}_e$ is the vector from sparker tip to eye at the arbitrary burst and $R(\theta_h, \theta_v, \theta_t)$ is the *rotation matrix* representing the rotation. The quantities θ_h, θ_v, and θ_t are

the values of the horizontal and vertical large head coil readings and the torsional reading of the smaller head coil. A full derivation of the form of the rotation matrix is given in the Appendix.

If equation (2) is applied to the on-biteboard case where $\mathbf{x}_e = \mathbf{x}_{eb}$, the vector $\mathbf{x}_{e0}$ can be written as

$$\mathbf{x}_{e0} = R^{-1}(\theta_{hb},\theta_{vb},\theta_{tb})\mathbf{x}_{eb} = R^{-1}(\theta_{hb},\theta_{vb},\theta_{tb})(\mathbf{r}_{eb} - \mathbf{r}_{hb}) \quad (3)$$

where θ_{hb}, θ_{vb}, and θ_{tb} are the head coil readings in the on-biteboard case and where equation (1) has been used.

Equation (1) also applies at an arbitrary RFM burst so that if $\mathbf{r}_e$ denotes the location of the sighting center of the eye at an arbitrary RFM burst and if $\mathbf{r}_h$ is the location of the sparker tip, then the vector from sparker tip to sighting center, $\mathbf{x}_e$ is given by

$$\mathbf{x}_e = \mathbf{r}_e - \mathbf{r}_h. \quad (4)$$

Combining equations (1)–(4) yields the fundamental equation which expresses the location of the sighting center of the eye at an arbitrary RFM burst completely in terms of *measured* quantities

$$\mathbf{r}_e = \mathbf{r}_h + R(\theta_h,\theta_v,\theta_t)R^{-1}(\theta_{hb},\theta_{vb},\theta_{tb})(\mathbf{r}_{eb} - \mathbf{r}_{hb}) . \quad (5)$$

The vector $\mathbf{r}_h$ represents the location of the sparker tip and θ_h, θ_v, θ_t are the head coil angle readings at an arbitrary burst of the RFM. The vectors $\mathbf{r}_{eb}$ and $\mathbf{r}_{hb}$ and the angles θ_{hb}, θ_{vb}, θ_{tb} are the on-biteboard sighting-center location, sparker-tip location, and head coil angle readings respectively.

Finding the Worktable coordinates of the line-of-sight vectors at an arbitrary RFM burst

The information needed to find line-of-sight vectors at an arbitrary RFM burst, in Worktable coordinates, consists of the sighting-center locations and the values of the vertical and horizontal eye coil angles [$\theta_v^{(e)}$ and $\theta_h^{(e)}$ respectively], at that burst. These angles are assumed to have already been corrected for placement offsets of the annuli (measured during mirror trials) and are referenced to the Worktable x-axis. This being the case, the vertical angle is the orthographic projection of a unit vector direction along the line-of-sight onto the xz-plane of the Worktable coordinate system. The horizontal angle is the orthographic projection of this unit vector onto the xy-plane of this coordinate system.

Figure 1(b) illustrates the meanings of the eye coil angles by showing the line-of-sight unit vector (denoted by $\mathbf{u}$) along with its orthographic projections onto the xz-and xy-planes. If the unit vectors along the x-, y-, and z-axes of the Worktable coordinate system are denoted by $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ respectively, then $\mathbf{u}$ can be expressed in terms of its components along these three axes as

$$\mathbf{u} = u_x\hat{\imath}_w + u_y\hat{\jmath}_w + u_z\hat{k}_w. \quad (6)$$

The components of $\mathbf{u}$ are also depicted in Fig. 1(b). Finding the line-of-sight unit vector in Worktable coordinates consists of expressing the three components of $\mathbf{u}$ in terms of the eye coil angles $\theta_v^{(e)}$ and $\theta_h^{(e)}$.

This is done by considering the right triangles **AOB** and **BOC** in Fig. 1(b). By the definition of tangent the following relations hold

$$\tan(\theta_v^{(e)}) = \frac{u_z}{u_x}, \ \tan(\theta_h^{(e)}) = \frac{u_y}{u_x}. \quad (7)$$

Since $\mathbf{u}$ is a *unit* vector, it follows that

$$u_x^2 + u_y^2 + u_z^2 = 1. \quad (8)$$

By combining equations (7) and (8) the line-of-sight unit vector can be expressed in terms of the eye coil angles as follows

$$\mathbf{u} = \frac{1}{D(\theta_h^{(e)},\theta_v^{(e)})}[\hat{\imath}_w + \hat{\jmath}_w\tan(\theta_h^{(e)}) + \hat{k}_w\tan(\theta_v^{(e)})], \quad (9)$$

where

$$D(\theta_h^{(e)},\theta_v^{(e)}) = [1 + \tan^2(\theta_h^{(e)}) + \tan^2(\theta_v^{(e)})]^{1/2}. \quad (10)$$

Finally, any point along the line-of-sight can be written as

$$\mathbf{r}(s) = \mathbf{r}_e + s\mathbf{u}, \quad (11)$$

where s is the linear distance from the sighting center to $\mathbf{r}$.

The binocular gaze point and the cyclopean line-of-sight

The cyclopean line-of-sight at an arbitrary RFM burst is defined as the line that passes through the midpoint of the subject's baseline (the line connecting the two sighting centers) and through the *binocular gaze point*. Binocular gaze point is defined to be the midpoint of the line which joins a point on the right eye's line-of-sight and a point on the left eye's line-of-sight, such that the distance between these two points is smaller than for any other such pair of points. It is easy to show that the line joining these two points will be perpendicular to the lines-of-sight of both eyes simultaneously (Edwards, Pizlo, Erkelens, Collewijn, Epelboim, Kowler, Stepanov & Steinman, 1994).

Let the lines-of-sight for the right and left eyes be parametrized as in equation (11), then

$$\mathbf{r}_r = \mathbf{r}_{re} + s_r\mathbf{u}_r, \quad (12)$$

$$\mathbf{r}_l = \mathbf{r}_{le} + s_l\mathbf{u}_l, \quad (13)$$

define arbitrary points on the lines-of-sight of the right and left eyes respectively. The vector locating the midpoint of the baseline is the average of the vectors locating the right-and left-eye sighting-centers

$$\mathbf{r}_{mid} = \frac{1}{2}(\mathbf{r}_{re} + \mathbf{r}_{le}). \quad (14)$$

The binocular gaze point is found by determining the pair of points, one on each line-of-sight, which are closer together than any other such pair. Each point on the right eye's line-of-sight corresponds to a unique value of s_r in equation (12) and each point on the left-eye's line-of-sight corresponds to a unique value of s_l in equation (13). Thus the pair of points defining the line whose midpoint is the binocular gaze point are found by minimizing the distance

function

$$d(s_r,s_l) = |\mathbf{r}_r(s_r) - \mathbf{r}_l(s_l)|. \tag{15}$$

The pair of points, $(s_r^{(m)}, s_l^{(m)})$, for which $d(s_r, s_l)$ is minimum can be used to express the vector that locates the binocular gaze point

$$\mathbf{r}_{bgp} = \frac{1}{2}[\mathbf{r}_r(s_r^{(m)}) + \mathbf{r}_l(s_l^{(m)})]. \tag{16}$$

Furthermore, these points, $(s_r^{(m)}, s_l^{(m)})$, can be written in terms of quantities already computed as follows

$$s_r^{(m)} = \frac{(\mathbf{b}\cdot\mathbf{u}_l)(\mathbf{u}_r\cdot\mathbf{u}_l) - (\mathbf{b}\cdot\mathbf{u}_r)}{1 - (\mathbf{u}_r\cdot\mathbf{u}_l)^2} \tag{17}$$

and

$$s_l^{(m)} = \frac{-(\mathbf{b}\cdot\mathbf{u}_r)(\mathbf{u}_r\cdot\mathbf{u}_l) - (\mathbf{b}\cdot\mathbf{u}_l)}{1 - (\mathbf{u}_r\cdot\mathbf{u}_l)^2} \tag{18}$$

where

$$\mathbf{b} = \mathbf{r}_{re} - \mathbf{r}_{le}. \tag{19}$$

The unit vector along the cyclopean direction is given by

$$\mathbf{u}_{cyc} = \frac{(\mathbf{r}_{bgp} - \mathbf{r}_{mid})}{|\mathbf{r}_{bgp} - \mathbf{r}_{mid}|}. \tag{20}$$

Finally, the line along the cyclopean line-of-sight can be parametrized similarly to the lines-of-sights of the right and the left eyes so that an arbitrary point on this line, a distance s_{cyc} from the midpoint of the subject's baseline is written as

$$\mathbf{r}(s_{cyc}) = \mathbf{r}_{mid} + s_{cyc}\mathbf{u}_{cyc}. \tag{21}$$

Cyclopean gaze-error with respect to a target on the Worktable

"Gaze-error" with respect to a given target was defined as the angle between a line-of-sight vector and a vector from the sighting-center to the target. The gaze-error could be calculated using either the right line-of-sight, the left line-of-sight, or the cyclopean line-of-sight. In case of the cyclopean line-of-sight, $\mathbf{r}_{mid}$ was used in place of the sighting-center. We found that when a subject fixated a target during our experiments, gaze-errors computed using the cyclopean line-of-sight were, on average, smaller than the gaze-errors computed using either the right or the left line-of-sight. For this reason cyclopean gaze-errors were used in the analyses.

The equation for cyclopean gaze-error with respect to target t is

$$\epsilon_{cyc} = \cos^{-1}(\mathbf{u}_{cyc}\cdot\mathbf{u}_t). \tag{22}$$

Unit vector $\mathbf{u}_t$ is the direction from the midpoint of the subject's baseline to the target given by

$$\mathbf{u}_t = \frac{(\mathbf{r}_{mid} - \mathbf{r}_t)}{|\mathbf{r}_{mid} - \mathbf{r}_t|} \tag{23}$$

where $\mathbf{r}_t$ is the vector representing the Worktable coordinates of the target t.

The current study is the first to use the complete MRFM setup and the sighting-center calibration procedure (described above). The RFM has been used previously to study free-headed subjects, but in all previous experiments the head coil apparatus consisted of only one coil, and therefore there was not enough information to calculate the location of the subject's line-of-sight (Collewijn *et al.*, 1992a,b). A more complete description of the of the MRFM and the calculations involved in processing its data are available in Edwards *et al.* (1994).

PROCEDURE

Two types of tasks were used. In the first task (tap), the subjects were asked to tap a sequence of targets located on the Worktable. Targets were rods (extending 2.3 cm above the surface of the Worktable) topped with colored LEDs. In the other task (look-only), the subjects were asked to look at a sequence of targets on the Worktable, without tapping them. Tapping and looking-only trials were run during separate sessions on separate days.

The sequences contained two, four or six targets. The order of targets was determined in one of two ways. In the pre-determined target-order condition, the subjects tapped or looked at targets in an order indicated by the colors of their LEDs, namely yellow, green, red, flashing-yellow, flashing-green, flashing-red. The flashing LEDs flickered at the rate of 10 Hz. The order of the colors was the same for the shorter sequences, i.e. the two-target sequence started with yellow and ended with green, and the four-target sequence started with yellow and ended with flashing yellow. In the self-selected target-order condition, the subjects followed any order they wanted, as long as each target was included exactly once. A given sequence was tested in a block of 10 trials, with the sequence repeated once on each trial. Before the start of each block, while the subject's eyes were closed, the experimenter placed the targets in the wells on the Worktable in the locations selected randomly by the computer. The configurations were selected so that no two targets were placed in adjoining wells. When the experimenter signaled that he was finished, the subject placed his right index finger on the home target and began the trial, when ready, by pressing a button held in his left hand. Then, he opened his eyes and proceeded to perform the looking or tapping sequence. Trial length was set to 4 sec (two target sequences), 6 sec (four target sequences) or 9 sec (six target sequences). At the end of each sequence in the tapping condition, the subject pressed the home target and closed his eyes. At the end of each sequence in the look-only condition, he looked at the home target and closed his eyes. The home target was the last target in each sequence, regardless of whether the target order was pre-determined or self-selected. The subject kept his eyes closed at all times when he was not performing a sequence.

All targets were visible continuously throughout the trial. The experiments took place in a well-lit room, permitting view of the walls and coil-frame in the background. Viewing was binocular and the subject's head was free to move. No instructions were given as to

how to move head, eyes, or arm. The instruction was to remain seated and perform as quickly as possible without missing targets or tapping or looking in an incorrect order.

The LEDs on top of the targets were 5 mm in diameter. The visual angle subtended by the LED depended on the location of the target and the position of the subjects head, which moved throughout the trial. The visual angles of the LEDs, therefore, ranged from about 0.25 to 0.75 deg.

RESULTS

Subjects reported that looking-only was more difficult than tapping and that they used different scanning patterns in each task

These differences were evident in the spontaneous comments made by the subjects as the data were collected. All four reported that tapping the targets was relatively easy and even fun, whereas sitting and looking at the targets in sequence seemed very unnatural, pointless, and required more effort. They were also convinced that they had used different scanning patterns in the two tasks. They reported that during tapping they looked at each target while they were learning the pattern during the first few trials in each 10-trial block, but once they were familiar with the pattern they reported that they often tapped targets without first looking at them. However, when they were only looking, they reported that they had followed the instructions and had looked at each target on each trial. Objective tests of our subjects' subjective reports were performed. The results of these tests are summarized in the following sections.

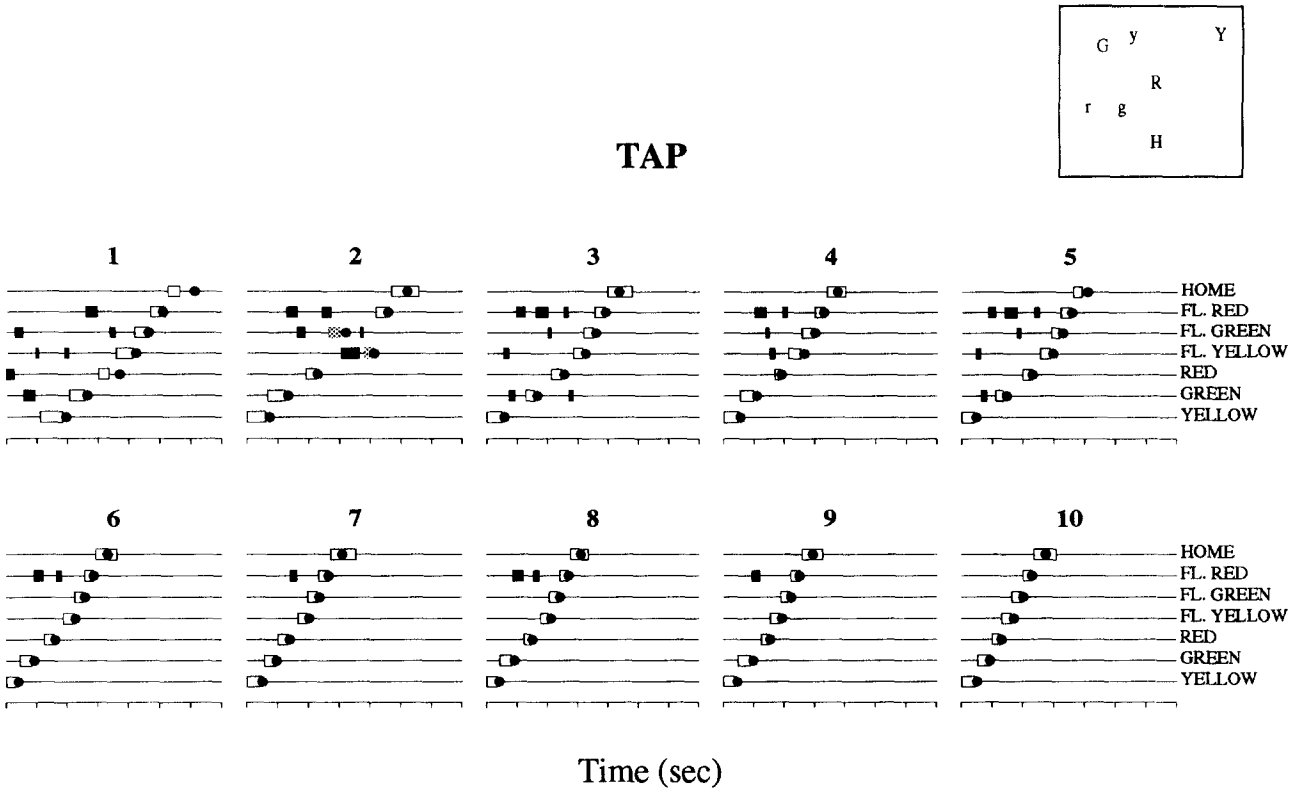
In the tapping task, subjects almost always looked at each target just before tapping it

After the subjects shifted gaze to a new target, they sometimes made several small saccades in the region on the Worktable within two grid spaces of the new target (recall that two grid spaces was the smallest separation between targets on the Worktable used in these experiments). Most of these saccades (about 80%) were corrective in that they reduced the distance between the cyclopean line-of-sight (defined in Method) and the target. These saccades and the intervals between them formed what we call a "looking episode". Henceforth, the term looking episode will be used to signify a period of time between the arrival of the cyclopean line-of-sight to a region within two grid spaces of a target and the beginning of the gaze-shift to a new target. We considered the small saccades made during a looking episode, which did not move the line-of-sight closer to a new target, to be part of the same looking episode.

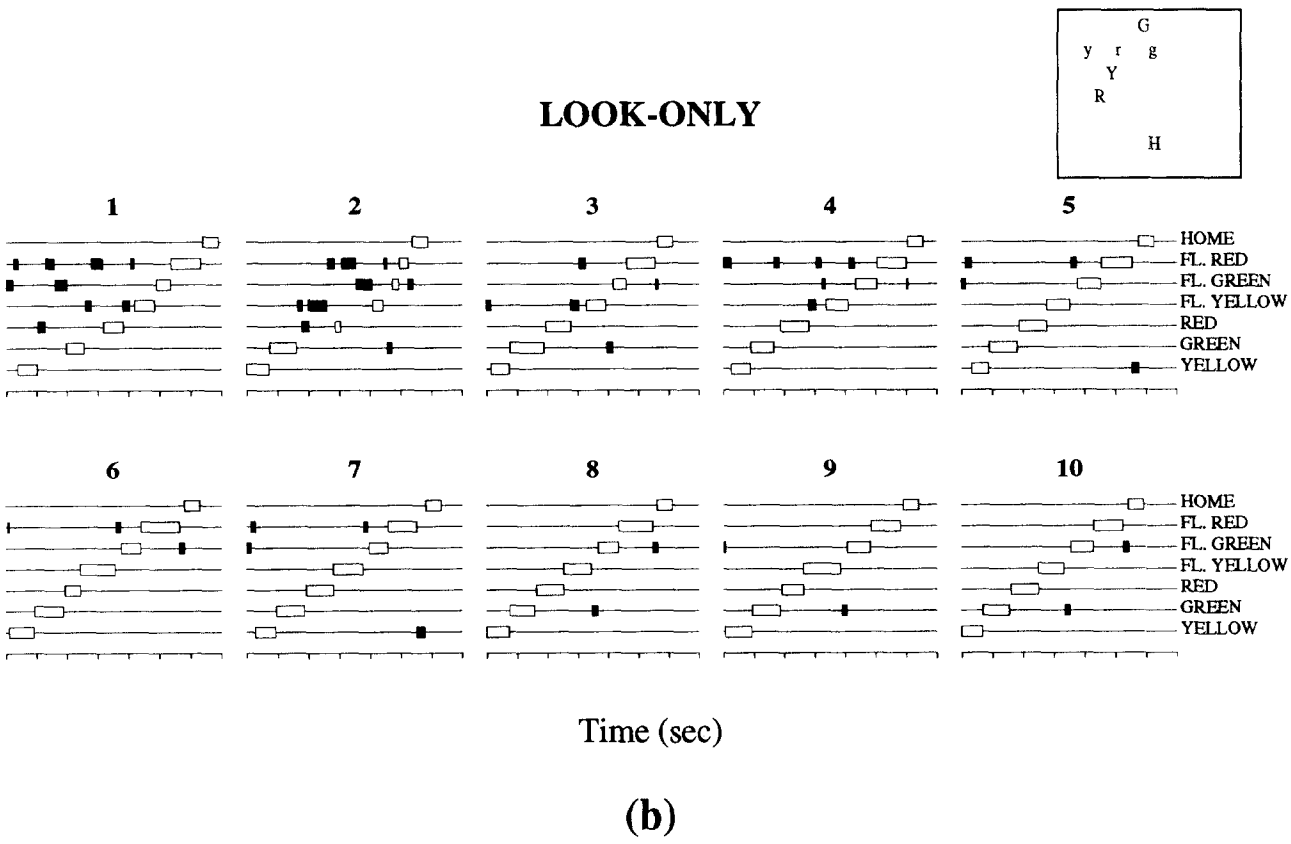
A new notation, called a "look/tap diagram", was developed to help visualize the events that took place as subjects performed looking and tapping sequences. These events included both looking episodes and taps. Figure 2 shows a representative set of look/tap diagrams for a block of tapping trials [Fig. 2(a)], and a block of looking-only trials [Fig. 2(b)]. The subject was CE, the target order was pre-determined, and there were six targets. The positions of the targets on the Worktable for each of these blocks of 10 trials are shown above and on the right of each set of diagrams.

Each diagram in Fig. 2 represents one repetition in the block of 10 trials with the ordinal position of the trial within the block numbered from 1 to 10 above each diagram. Time is shown on the abscissa. Each horizontal line above the abscissa represents one of the targets and is labeled with that target's color name. Each rectangular box represents one looking episode. The widths of the rectangular boxes signify the durations of these looking episodes. The solid circles show when the subject tapped each target. Looking episodes were divided into two classes. The open rectangular boxes represent looking episodes made near targets when they were looked at in the correct order. Henceforth, these looking episodes, represented by the open rectangular boxes, will be called "sequence episodes". Solid boxes represent looking episodes that were out of the correct target sequence and not followed by a tap. Henceforth, these looking episodes, represented by solid rectangular boxes, will be called "search episodes". There was a third and very infrequent class of looking episodes, i.e. looking episodes that were followed by taps, but the taps were not in the correct target order. These erroneous episodes are shown with shaded boxes. Repetition 2 in Fig. 2(a) shows two such looking episodes. They are distinguished from regular sequence episodes because this repetition contained an error in the tapping order.

First consider the representative block of 10 tapping trials [Fig. 2(a)]. During the first five repetitions the subject often looked at targets without tapping them (solid boxes). These search episodes were not part of the correct sequence, and presumably, CE used them to search for the next target in the sequence. Every time CE did find the correct target in the sequence, he always looked at it just before tapping it (each solid circle is immediately preceded by a sequence episode, i.e. an open box on the same line). In the second part of the block (after the fifth repetition), CE looked at targets in the correct order almost all of the time, i.e. there are very few solid boxes in the second row of Fig. 2(a). It is clear that CE continued looking at each target immediately before tapping it even after he had learned the pattern and no longer had to search for each individual target. This strategy of looking at each target just before tapping it was used on virtually all tapping trials (on all 347 trials for CE, on 434 out of 463 trials for HC, 357 out of 360 trials for ZP, and on 389 out of 402 trials for RS). This look-before-tap strategy was used regardless of the number of targets in the sequence, and regardless of whether the order of the targets was pre-determined or self-selected. It persisted even in the rare trials that contained tapping-order errors. One of these is shown in repetition 2 in Fig. 2(a),where CE reversed the required order when he looked at and tapped the flashing green target before he tapped the flashing yellow target. The order should have been flashing yellow then flashing green. The fact that tapping errors were always



(a)



(b)

FIGURE 2. Look/tap diagrams for (a) a 10-trial block of tapping trials and (b) a 10-trial block of looking-only trials (b). Each diagram represents one repetition with the same target configuration. The repetitions are numbered 1–10 above each diagram. The position of targets on the Worktable for each block are shown in the rectangle, representing the Worktable above each block of diagrams. Letters inside the rectangle signify the colors of the six LED targets and a home button: Y, yellow; G, green; R, red; y, flashing yellow; g, flashing green; r, flashing red; H, home. Time is shown on the abscissa. Each horizontal line above the abscissa represents one target, labeled with its color on the right. Boxes and dots on each horizontal line show when the subject looked at or tapped that target. Open boxes show sequence episodes, solid boxes show search episodes, shaded boxes show looking episodes preceding tapping errors (see text). Widths of boxes show durations of looking episodes. Solid circles show the times of the taps. See the text for full explanation.

accompanied by looking errors suggests that these errors arose from improper target selection, not from poor coordination of individual eye/head/arm motor programs.

An examination of the rare trials (45 of the 1572 collected for all subjects) in which a subject *did* skip looking at one of the targets showed that in 39 of these trials, skipping occurred when two of the targets were located within 2 grid spaces on the Worktable, usually in the same column. When this happened, the subjects, especially HC, who skipped targets most frequently (6% of his trials), tended to look at the region between the two nearby targets and then tapped both targets in the correct order without looking at each before tapping it.

Now, consider the look/tap diagrams for a representative block of looking-only trials [Fig. 2(b)]. Here, as in the tapping block shown just above, there were many search episodes during the first five repetitions (solid boxes). The number of search episodes also decreased with repetitions just as it did in the tapping sequences.

Recall that the subjects reported that when they were tapping, they often skipped looking at targets, particularly during the second half of the block after they had become familiar with the configuration of the targets. This subjective impression was counter-factual. Subjects *did* look at each target before they tapped it. Gaze accuracy during sequence and search episodes are considered next.

Gaze-errors during sequence episodes were smaller during looking-only than during tapping, but there were no differences in the sizes of gaze-errors during search episodes

Figure 3 shows mean cyclopean gaze-errors for sequence [Fig. 3(a)] and search [Fig. 3(b)] episodes during looking-only (open bars) and tapping (shaded bars). Recall that cyclopean gaze-errors were smaller than the gaze-errors for either eye. The mean gaze-errors in Fig. 3 were computed after all corrective saccades had been completed and, therefore, represent the minimum mean gaze-errors observed during looking episodes.

Mean gaze-errors for sequence episodes in Fig. 3(a) ranged from 1.2 to 2.7 deg—approximately the same size as the fovea (Polyak, 1941, p. 198 gives the human foveal radius as 2.5 deg). Gaze-errors for sequence episodes were smaller during looking-only than during tapping. These differences were not large, but they were statistically reliable [$P < 0.001$: CE, $t(2764) = 6.3$; HC, $t(2315) = 8.8$; ZP, $t(2990) = 11.2$; RS, $t(2788) = 24.1$]. The mean gaze-errors shown in Fig. 3 are for pre-determined target order. Mean gaze-errors of each of the four subjects for self-selected target order are summarized in the rightmost column of Table 1. They were very similar to the gaze-errors observed when the target order was pre-determined.

There are two possible sources for the differences in the size of sequence episode gaze-errors in the two tasks. First, consider the initial gaze-errors, i.e. the gaze-errors computed at the start of the looking episode immediately following the end of the first gaze-shift into the region near the first target. These initial gaze-errors were smaller

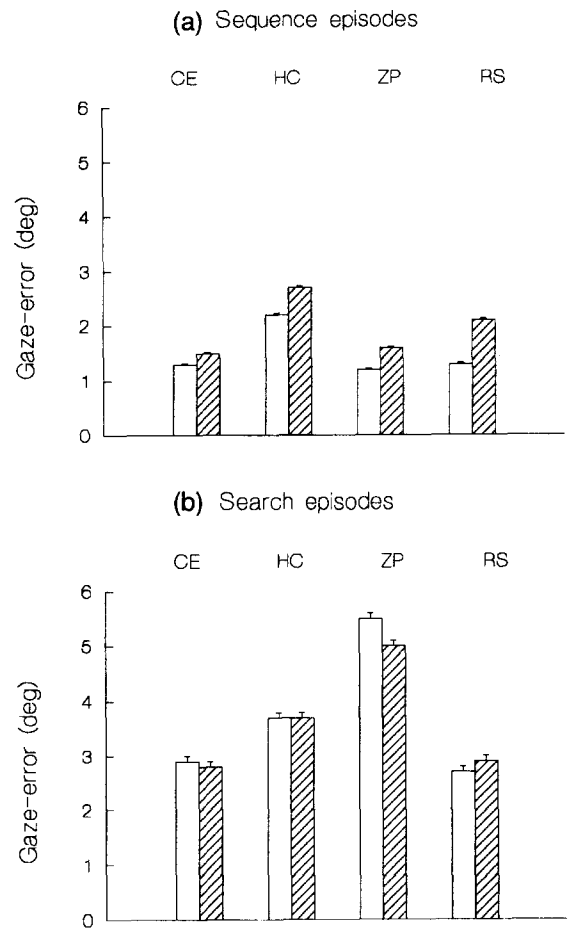


FIGURE 3. Mean gaze-errors (deg) during sequence (a) and search (b) episodes, for looking-only (open bars) and tapping (shaded bars) in pre-determined target order. Each bar is based on 919–1396 sequence episodes in (a) and on 247–552 search episodes in (b). Error bars show 1 SE.

during looking-only than during tapping (see Table 1). They were, however, on average, 15% of the preceding gaze-shift in both tasks, and gaze-shifts tended to be larger during tapping than during looking-only simply because subjects moved closer to the table when they tapped. This increased the angular separations between the targets, which led to larger initial gaze-errors because the size of the error was proportional to the size of the preceding gaze-shift and gaze-shifts during the tapping task were larger. Second, corrective saccades made during some sequence episodes in the looking-only task tended to be somewhat more effective in the sense that they left smaller gaze-errors. Data supporting these statements about initial and final gaze-shift accuracy during sequence episodes are summarized in Table 1.

About 22–40% of tapping sequence episodes and 15–24% of looking-only sequence episodes contained saccades (i.e. saccades in addition to the large gaze-shift that is made at the beginning of each looking episode). Most of these saccades (over 75%) were corrective. Corrective saccades were made more frequently during tapping than during looking-only, probably because during tapping initial gaze-errors were larger (Table 1). Initial gaze-errors shown in Table 1 range from 2.0 to

TABLE 1. Characteristics of sequence episodes

	Target-order	Task	No. of sequence episodes (N)	No. sequence episodes with saccades (% of N)	No. of sequence episodes with corrective saccades (% of N)	Mean (SD) initial gaze-error (deg)	Mean (SD) amount of correction (deg)	Mean (SD) time for correction (deg)	Mean final gaze-error (deg)
Subject CE	Pre	Look	919	138 (15%)	125 (14%)	2.0 (1.7)***	3.2 (2.1)	206 (95)***	1.3 (0.7)***
		Tap	916	357 (40%)	323 (35%)	2.6 (1.8)	2.9 (1.8)	165 (56)	1.5 (0.8)
	Self	Look	465	69 (15%)	64 (14%)	2.1 (2.1)	4.3 (3.2)***	226 (105)***	1.2 (0.5)***
		Tap	466	123 (26%)	113 (24%)	2.2 (1.6)	2.9 (1.7)	174 (77)	1.4 (0.6)
Subject HC	Pre	Look	921	111 (12%)	95 (10%)	2.7 (1.7)***	3.1 (2.3)	259 (200)	2.2 (1.0)***
		Tap	1361	297 (22%)	200 (15%)	3.2 (2.4)	3.0 (2.7)	244 (108)	2.7 (1.7)
	Self	Look	413	62 (15%)	43 (10%)	3.1 (1.7)***	4.6 (3.5)**	275 (110)	2.4 (0.8)***
		Tap	433	127 (29%)	66 (16%)	3.7 (3.0)	3.0 (2.2)	294 (299)	2.7 (1.8)
Subject ZP	Pre	Look	950	231 (24%)	185 (19%)	2.4 (2.5)***	4.1 (3.5)*	335 (255)***	1.3 (0.7)***
		Tap	1101	360 (32%)	293 (27%)	2.7 (2.2)	3.4 (2.8)	236 (126)	1.6 (1.1)
	Self	Look	473	79 (17%)	52 (11%)	1.6 (1.6)***	3.1 (3.3)	221 (121)	1.2 (0.8)***
		Tap	468	145 (31%)	120 (26%)	2.6 (2.5)	3.7 (2.9)	223 (91)	1.6 (0.9)
Subject RS	Pre	Look	949	193 (20%)	159 (17%)	2.0 (1.7)***	3.4 (2.0)*	213 (97)	1.1 (0.7)***
		Tap	1038	243 (23%)	187 (18%)	2.8 (1.7)	2.9 (1.8)	195 (89)	2.2 (1.1)
	Self	Look	432	97 (22%)	64 (15%)	2.3 (2.0)**	4.1 (2.6)**	274 (158)***	1.6 (1.0)***
		Tap	371	92 (25%)	73 (20%)	2.5 (1.7)	2.9 (2.2)	203 (80)	.9 (1.0)

Asterisks show results of t-tests comparing look-only and tap means (*$P < 0.05$; **$P < 0.01$; ***$P < 0.001$). Mean initial and final gaze-errors were computed for all sequence episodes. Mean correction and mean time for correction values were computed only for sequence episodes that contained corrective saccades.

3.7 deg. They are not much larger than the final gaze-errors, which are shown in the rightmost column. The mean amount of the correction of the initial gaze-errors, shown in the third column from the right, ranged from 2.9 to 4.3 deg. These corrections seem to be excessive given the *initial* and *final gaze-errors* summarized in Table 1, but recall that corrections occurred in fewer than 40% of sequence episodes. This means that on most of the sequence episodes, the initial gaze-shift was sufficiently close to the target to allow the subject to avoid subsequent corrections. In sequence episodes that did contain corrective saccades, mean times spent making these saccades ranged from 165 to 335 msec. These times were 10–99 msec longer during looking-only than during tapping, so spending more time in each sequence episode during looking-only served to allow more complete gaze-error correction in this task. All subjects showed this tendency, but the effect of the type of task on the mean amount of correction reached significance only for some subjects and target-order conditions (see Table 1).

Looking closely at targets was less important during search. Search episode gaze-errors (after final corrections) ranged from 2.7 to 5.4 deg [Fig. 3(b)]. However, 14% of search episodes contained small saccades, 75% of which were corrective. This suggests that the subjects did not scan the Worktable randomly, but were trying to look near the positions of the targets. Furthermore, on over 95% of search episodes, the line-of-sight was within 2 grid spaces of one of the targets after the initial gaze-shift. This means that the subjects rarely looked at regions on the Worktable that did not contain targets. Instead, they detected the presence of a target in the periphery and then looked at it more closely to determine or to confirm its color.

Global eye movement characteristics examined so far were quite similar in both tasks. Namely, subjects looked at each target in both tasks and the differences in gaze-errors were small during sequence episodes and non-existent during search episodes. Large differences in performance, however, were observed when the times required to complete looking and tapping sequences and effects of practice on completion time were compared. These results are presented next.

Subjects got faster with repetition during tapping, but not during looking-only

Figures 4 and 5 show mean times to complete looking-only (○) and tapping (●) sequences plotted as a function of repetition. Data for pre-determined target order are shown in Fig. 4, and data for self-selected target order are shown in Fig. 5. In both figures, data for six- (top row), four- (middle row) and two-target sequences (bottom row) are shown. Looking-only and tapping sequences were considered completed at the end of the gaze-shift that brought the line-of-sight to the home target. The additional time required to tap the final home target in the tapping task was not included because this would have artificially prolonged the duration of the tap sequences by an average of 404 msec (SD = 140 msec) relative to the duration of the looking-only sequences. In order to compare equivalent oculomotor tasks, trials in which the number of sequence episodes was *not* equal to the number of targets were excluded from all subsequent analyses. These were the rare tapping trials in which subjects did not look at each target before tapping it.

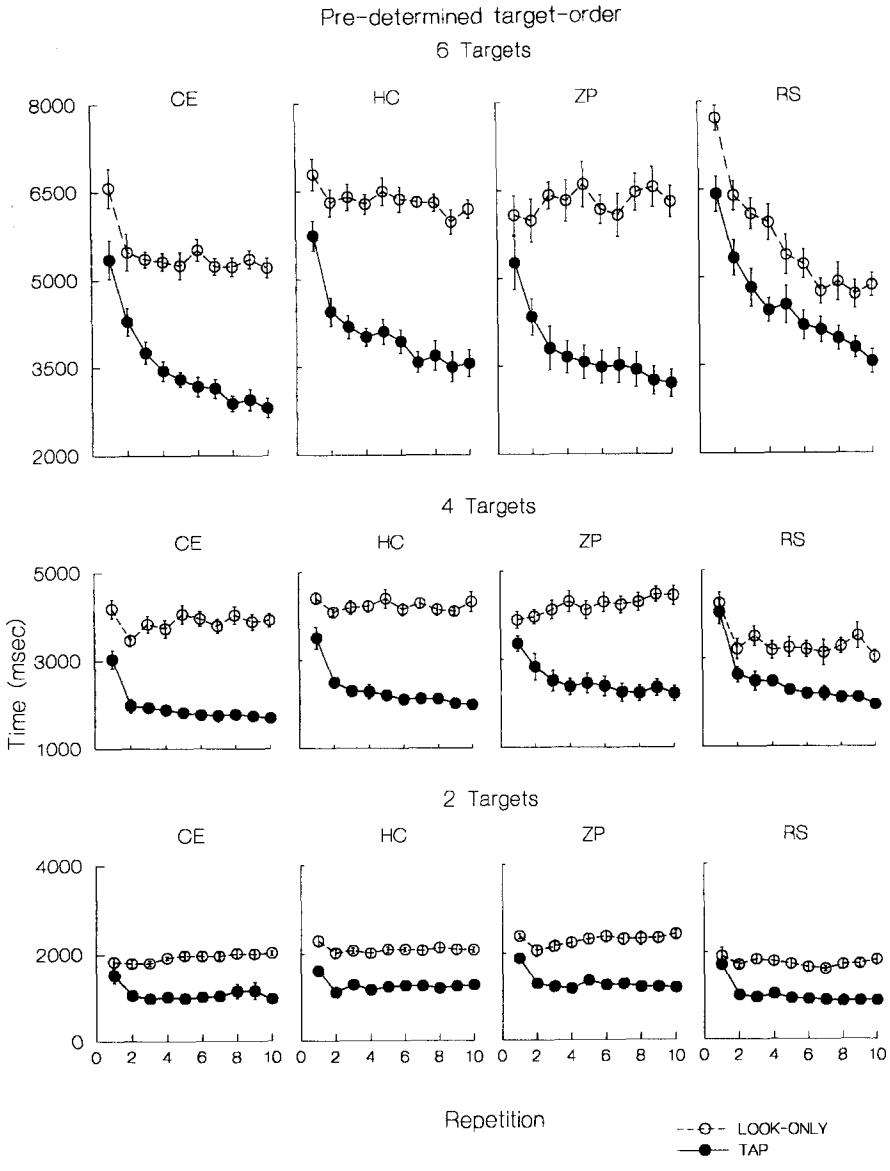


FIGURE 4. Mean times (msec) to complete tapping (●) and looking-only (○) sequences for pre-determined target order, plotted as a function of repetitions. Data for four subjects are shown in columns labeled CE, HC, ZP, and RS. Data for sequence lengths are shown in rows labeled 6 targets, 4 targets, and 2 targets. Each datum point is based on 7–12 trials. Error bars show ±1 SE.

Trials in which tapping-order errors occurred, or in which subjects failed to complete the tapping sequence were also excluded. Also excluded were looking-only trials in which the subject did not look at each target in the correct sequence when the order was pre-determined, or did not look at each target at least once when the order was self-selected. Such trials were rare, so despite all of these exclusions only 3% of the available trials were removed from consideration. Looking-only trials that had search episodes embedded in the correct sequence, *were* included.

First consider the pre-determined target-order condition, shown in Fig. 4. Two striking differences between looking-only and tapping plots can be observed. First, more time was required to complete looking-only sequences throughout the 10 repetitions. Second, times to complete the tapping sequences decreased considerably by the 10th repetition, especially for sequence lengths of four and six targets. Furthermore, with six targets, the subjects were still getting faster even after the 10th repetition. Looking-only, on the other hand, showed very little decrease in time for subjects CE, HC, and ZP. Among these three subjects, the biggest time difference between the first and the last looking-only repetitions was 1300 msec (20%) shown by CE with six targets. Only one subject, RS, showed similar effects of repetition in both looking-only and tapping tasks. RS improved when he practiced with both four-and six-target configurations. Note, however, that RS still required more time (>1 sec) to complete looking-only sequences than tapping sequences, even by the 10th repetition. Moreover, with six targets, RS's looking-only times reached steady-state after the seventh repetition, whereas he was still getting faster after 10 repetitions of tapping. All subjects were still improving in the six-target condition of the tapping task at the end of each 10-trial block—asymptotic performance had not been reached. This means that the time differences between tapping and looking-only with six targets would have been even greater if the subjects had

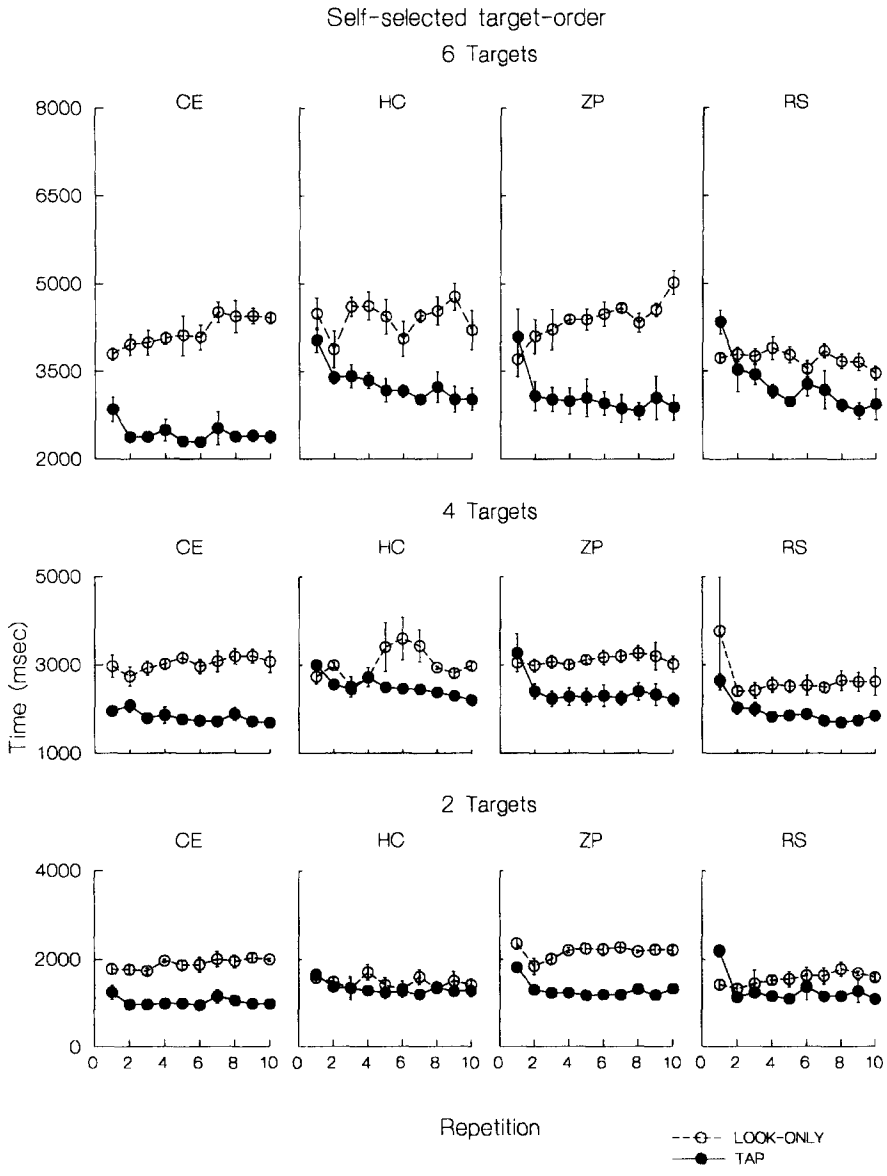


FIGURE 5. Mean times (msec) to complete tapping (●) and looking-only (○) sequences for self-determined target order, plotted as a function of repetition. Data for four subjects are shown in columns labeled CE, HC, ZP, and RS. Data for sequence lengths are shown in rows labeled 6 targets, 4 targets, and 2 targets. Each datum point is based on 3–4 trials. Error bars show ±1 SE.

been allowed more repetitions with each six-target configuration.

Effects of task (looking-only vs tapping) were statistically significant for all subjects [$P < 0.001$: CE, $F(1,402) = 1476.1$; HC, $F(1,508) = 1771.0$; ZP, $F(1,461) = 929.0$; RS, $F(1,431) = 322.2$]. In addition, all subjects except RS showed significant interactions between task and repetition [$P < 0.001$: CE, $F(9,402) = 6.4$; HC, $F(9,508) = 6.3$; ZP, $F(9,461) = 6.8$]. For RS the interaction between repetition and task was not significant—he decreased his time in the same manner in both tasks [$P > 0.3$, $F(9,431) = 1.1$].

Now consider Fig. 5, which plots times to complete the sequence as a function of repetition for the self-selected target-order condition. As expected, when target order was self-selected, subjects did not improve as much with practice as they did when target order was pre-determined. Without the order constraint, the task became easier and there was little room for improvement or benefit from repetition. Tapping continued to be faster than looking-only, but the differences in performance between the two task was diminished. Note that removing the target-order constraint affected looking-only and tapping tasks differently. Tapping times were only affected throughout the entire block when six targets were used. With fewer targets (two and four) effects were only observed during the first two repetitions. After the first two repetitions, target order had no reliable effect on tapping times with two and four targets [CE, $P > 0.5$, $F(1,172) = 0.4$; HC, $P > 0.2$, $F(1,185) = 1.4$; ZP, $P > 0.5$, $F(1,191) = 0.4$; RS, $P > 0.09$, $F(1,154) = 2.7$]. Looking-only, on the other hand, benefitted more from removing the order constraint. The effect of target order was significant for looking-only even during the last two repetitions [CE, $P < 0.001$, $F(1,58) = 31.5$; HC, $P < 0.002$, $F(1,57) = 12.5$; ZP, $P < 0.001$, $F(1,60) = 56.3$; RS, $P < 0.001$]. Despite this interaction between target order and task, the time difference between

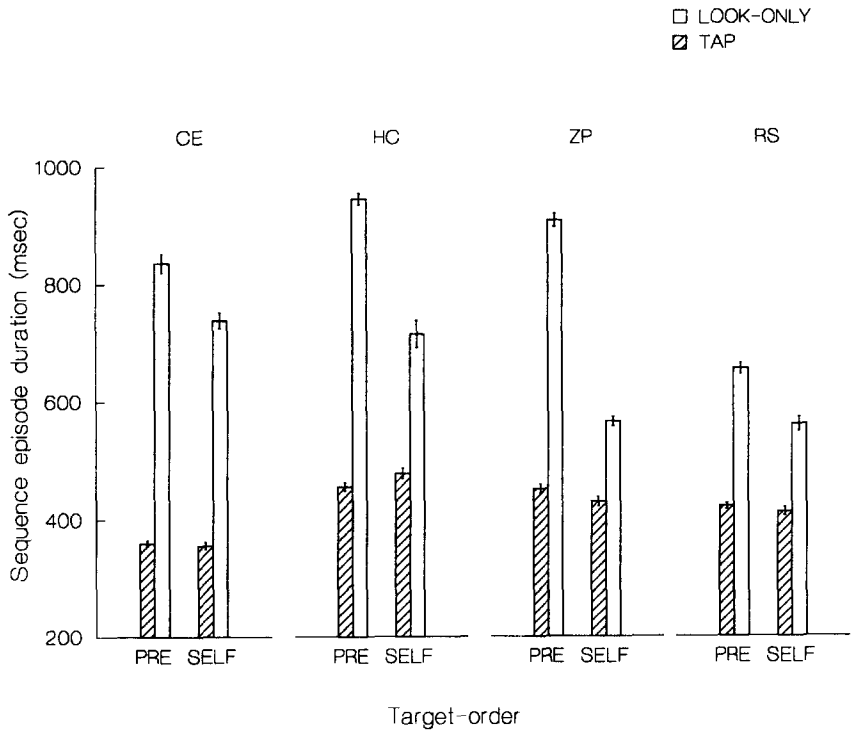


FIGURE 6. Mean sequence episode durations (msec) for pre-determined (PRE) and self-selected (SELF) target order conditions. Data for tapping are shown with shaded bars, and data for looking-only are shown with open bars. Each bar is based on 371–1361 sequence episodes. Error bars show ± 1 SE.

looking-only and tapping, even during the last five repetitions with self-selected target order, was substantial, at least with four and six targets [143–966 msec with two targets; 789–1358 msec with four targets; 586–1995 msec with six targets], and statistically significant [$P < 0.001$: CE, $F(1,87) = 708.2$; HC, $F(1,71) = 63.7$; ZP, $F(1,88) = 354.9$; RS, $F(1,72) = 104.9$].

The finding that looking-only sequences took more time to complete and did not benefit as much from practice (Figs 4 and 5) agrees with the subjects' impressions that looking-only was the more difficult task. This outcome was not expected nor was it intuitively obvious *a priori* because on theoretical grounds tapping should be harder. There is so much more to do. When tapping, the subjects had to aim for and tap relatively small targets, coordinating the movements of the eyes, the head, the arm, and often the upper torso as well. When looking-only, all they had to move were their eyes, and possibly their head if they were so inclined. Wide-ranging speculation about reasons for the result obtained is tempting, but we have data to support only one reason. Namely, the relative advantage of the tapping task came about, at least in part, because the positions and movements of the hand during tapping helped subjects remember the locations of the targets or the oculomotor commands that should be used to look at them. This idea is consistent with our observations that looking-only: (i) did not get faster with practice and (ii) benefitted more than tapping from the removal of the target-order constraint. Difficulties in remembering target locations or appropriate oculomotor commands could manifest themselves in either the time required to retrieve the next target position from memory, or in more time devoted to visual search of the Worktable during the looking-only task. Sequence episode durations will be examined next to determine whether durations could provide clues as to the use of memory in both tasks.

Sequence episodes lasted longer during looking-only than during tapping

Figure 6 shows mean durations of sequence episodes during looking-only and tapping for pre-determined and self-selected target order. Data were taken from the last five repetitions in a block, after most of the learning had been completed. Data were combined over the different sequence lengths because sequence length effects, were much smaller than the effects of the type of task—the effects of interest in this section.

When target order was pre-determined, sequence episode durations during looking were on average 476, 489, and 456 msec (45–55%) longer than the sequence episodes during tapping for subjects CE, HC, and ZP, respectively. For RS, the only subject who got faster as he looked repeatedly at a given target configuration, the difference in duration between looking-only and tapping sequence episodes was less, only 234 msec (24%). The effect of type of task was statistically significant for all subjects [$P < 0.001$: CE, $t(923) = 29.2$; HC, $t(1164) = 42.2$; RS, $t(1000) = 23.8$; ZP, $t(1048) = 34.1$]. When target order was self-selected, the differences between looking-only and tapping became smaller for subjects HC, ZP, and RS (135–238 msec, or 21–26%). For subject, CE, sequence episodes lasted 50% longer during looking-only than during tapping, regardless of target order. The difference in sequence episode duration between looking-only and tapping with self-selected

target order was statistically significant for all subjects [$P < 0.001$: CE, $t(459) = 26.0$; HC, $t(424) = 9.6$; ZP, $t(470) = 11.6$; RS, $t(401) = 10.2$]. These results are consistent with the observations about the total time required to complete looking-only and tapping sequences described above and shown in Figs 4 and 5. Note that even the smallest differences in sequence episode durations between looking-only and tapping, shown in Fig. 6, were larger than the extra time taken by corrective saccades, which occurred on more than 20% of the trials (see Table 1 and section on gaze-errors above). This means that the difference in sequence episode duration between looking-only and tapping *cannot* be accounted for by the assumption that the subjects took more time in order to look at the targets more accurately during looking-only.

The results summarized in Fig. 6 show that subjects dwelled longer around targets when they were looking-only than when they were tapping. What were they doing? We know that they did not spend all of the extra time making saccades in the target region, corrective or otherwise. Possibly they were searching memory for the next target location or motor command. We turned next to an analysis of visual search characteristics as a first step towards getting a handle on memory search because visual search could give some indication about how memory was used in both tasks. If target configurations were more difficult to remember during looking-only, more search episodes may have been required and the number of search episodes may not go down with practice. Effects of practice on the number of search episodes is considered next.

Practice caused the number of search episodes to be reduced at the same rate in looking-only and tapping tasks

Figure 7 plots the mean number of search episodes as a function of repetition when target order was pre-determined. The data for six, four, and two targets are

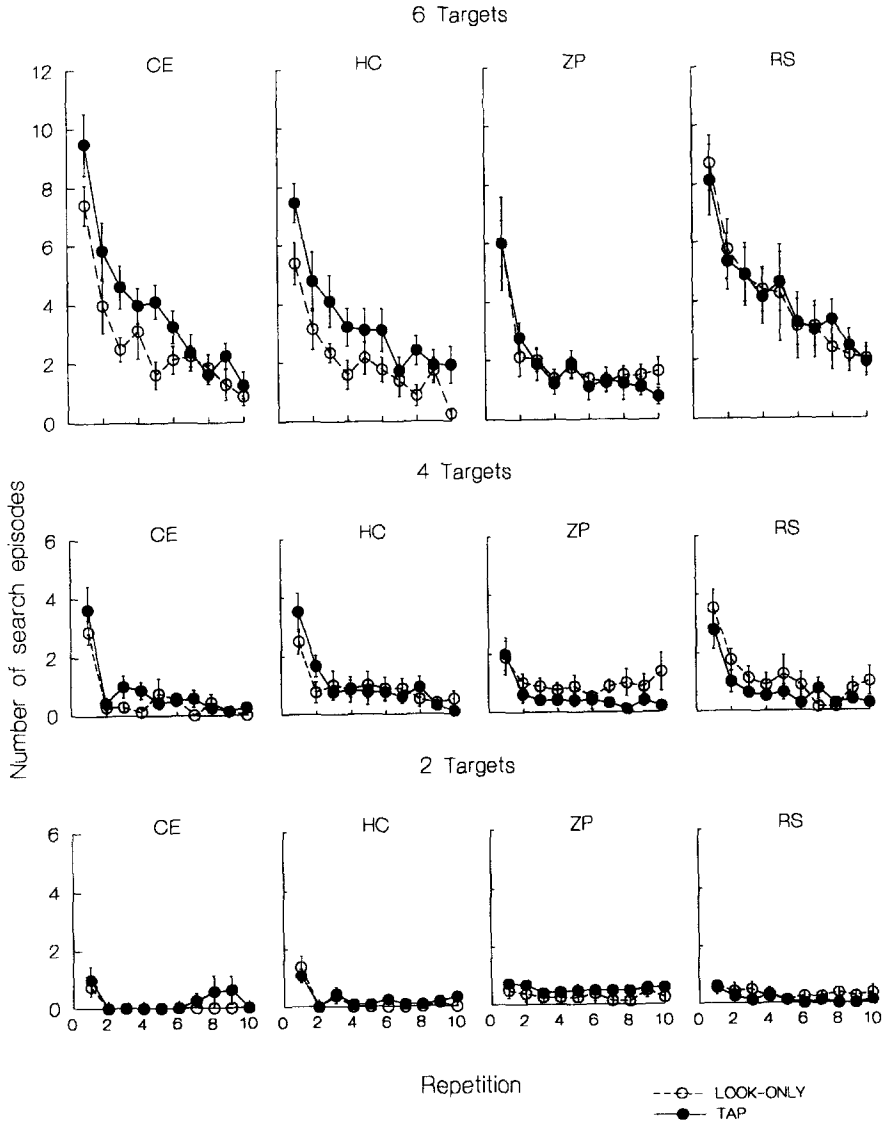


FIGURE 7. Mean number of search episodes in tapping (●) and looking-only (○) sequences plotted as a function of repetition for pre-determined target order. Data for four subjects are shown in columns labeled CE, HC, ZP, and RS. Data for sequence lengths are shown in rows labeled 6 targets, 4 targets, and 2 targets. Each datum point is based on 7–12 trials. Error bars show ±1 SE.

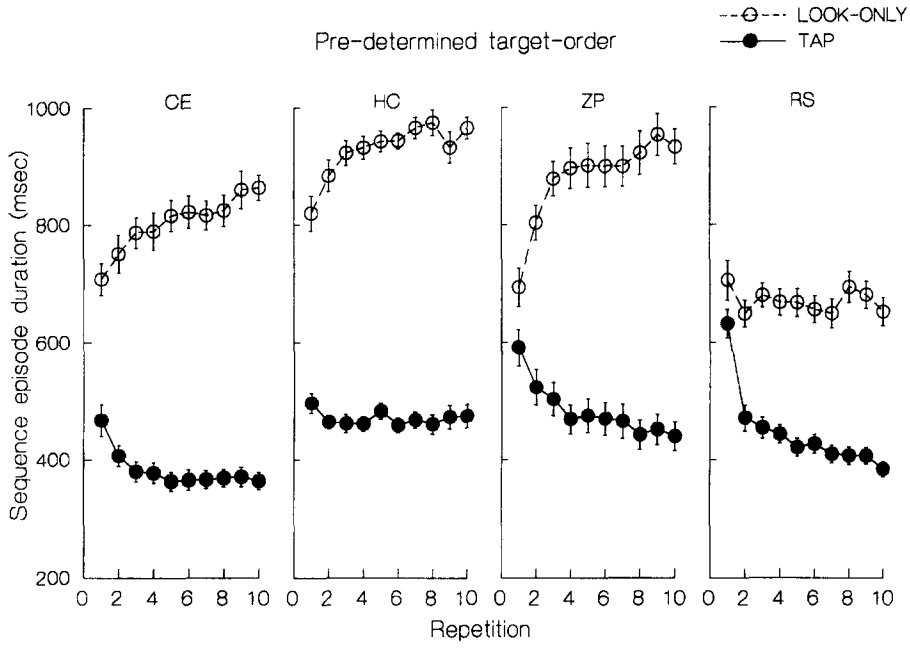


FIGURE 8. Mean sequence episode duration (msec) for tapping (●) and looking-only (○) with pre-determined target order plotted as a function of repetition. Each datum point is based on 22–35 trials. Error bars show ±1 SE.

shown. There were very few search episodes when target order was self-selected, so our analyses were confined to the pre-determined target-order condition.

The biggest effect on the number of search episodes in both tasks is the effect of repetition. The number of search episodes went down as the subjects repeatedly looked at or tapped the same target configuration. The effect was the biggest with six targets, and, at least for subjects CE and RS the number of search episodes was still decreasing after 10 repetitions. This suggests that in both tasks, target locations were being committed to memory, and, therefore, fewer search episodes were required with repetition. Two of the four subjects, CE and HC, showed reliable main effects of the type of task on the number of search episodes, with fewer search episodes during looking-only than during tapping [$P < 0.001$: CE, $F(1,409) = 26.2$; HC, $F(1,511) = 20.0$]. The largest difference was observed with the six-target configuration. It amounted to about one *more* search episode during tapping than during looking-only. For the other two subjects the number of search episodes was not significantly different between looking-only and tapping [RS, $P > 0.2$, $F(1,433) = 1.2$; ZP, $P > 0.2$, $F(1,463) = 1.5$]. The effect of repetition on the number of search episodes was the same in both tasks. The solid and dashed curves in Fig. 7 have the same shape, and the interaction between the type of task and repetition was not significant [CE, $P > 0.5$, $F(9,409) = 0.8$; HC, $P > 0.6$, $F(9,511) = 0.8$; ZP, $P > 0.8$, $F(9,463) = 0.5$; RS, $P > 0.9$, $F(9,433) = 0.2$].

In summary, there was no difference in the effect of practice on the number of search episodes between looking and tapping. This result was unexpected in view of the observed effects of practice on the speed of performance (Fig. 4). The number of search episodes, and therefore the total number of looking episodes decreased as a function of repetition in both looking-only and tapping tasks (Fig. 7). The relative time to perform these looking episodes, however, decreased only during tapping (Fig. 4). This implies that when the subjects were only looking, durations of sequence episodes increased as the number of search episodes decreased. The effect of repetition on duration of sequence episodes is considered next.

Duration of sequence episodes increased with repetitions during looking-only, but decreased with repetition during tapping

Figure 8 plots mean sequence episode durations for looking-only (dashed lines) and tapping (solid lines) as a function of repetition. The data are for pre-determined target order. With self-selected target order the effects of repetition were in the same direction, but smaller. The data for different sequence lengths were combined because the largest effect of sequence length on sequence episode duration was about 100 msec—much smaller than the effects of the type of task and repetition, the effects of interest in this section.

Plots in Fig. 8 show that tapping sequence episode duration *decreased* with repetition, whereas looking-only sequence episode duration *increased* with repetition. Examination of individual blocks of trials showed that during looking-only, as the number of search episodes decreased with repetition, the duration of sequence episodes increased, causing the total time to complete a looking-only sequence to remain constant or increase. *Thus, as the subjects relied more on memory and less on visual search, accessing the next component in the sequence became more difficult during looking-only than during tapping.*

This phenomenon helps explain why RS increased his speed with repetition during looking-only, while the other

subjects did not. RS had the largest number of search episodes, and on many blocks, especially with the six-target configuration (Fig. 6), he was still searching even after 10 repetitions. Thus, he did not rely on memory as much as the other subjects and was able to get faster by continuing to search the display throughout the 10 trials block of trials. It would be nice to know whether this strategy, which allowed RS to improve with repetition when he was only looking, represented wisdom on the part of our oldest and most experienced subject, or whether RS had no choice but to continue to search because his short-term memory had become less reliable with age. These are not, of course, mutually exclusive alternatives and we can offer no preference because our present data does not allow us to choose between them.

The analyses so far suggest that during sequence episodes three of our four subjects spent at least some of their time retrieving the next target location or oculomotor command from memory. The process of memory retrieval took more time during looking-only than during tapping. When memory failed, or when the subject chose not to rely on memory, visual search was used. In the next section, we compare the durations of sequence and search episodes in the two tasks.

Search acted as an independent process embedded in both looking-only and tapping tasks

Figure 9 shows the distributions of durations of looking episodes of different types. Duration is on the abscissa with bin widths set to 30 msec. The proportion of looking episodes falling in each bin is shown on the ordinate. Distributions of sequence episodes are shown with circles and search episodes with triangles. The solid symbols show data for the tapping task and the open symbols for the looking-only task. Only data for pre-determined target order are plotted. Figure 9 shows that the different

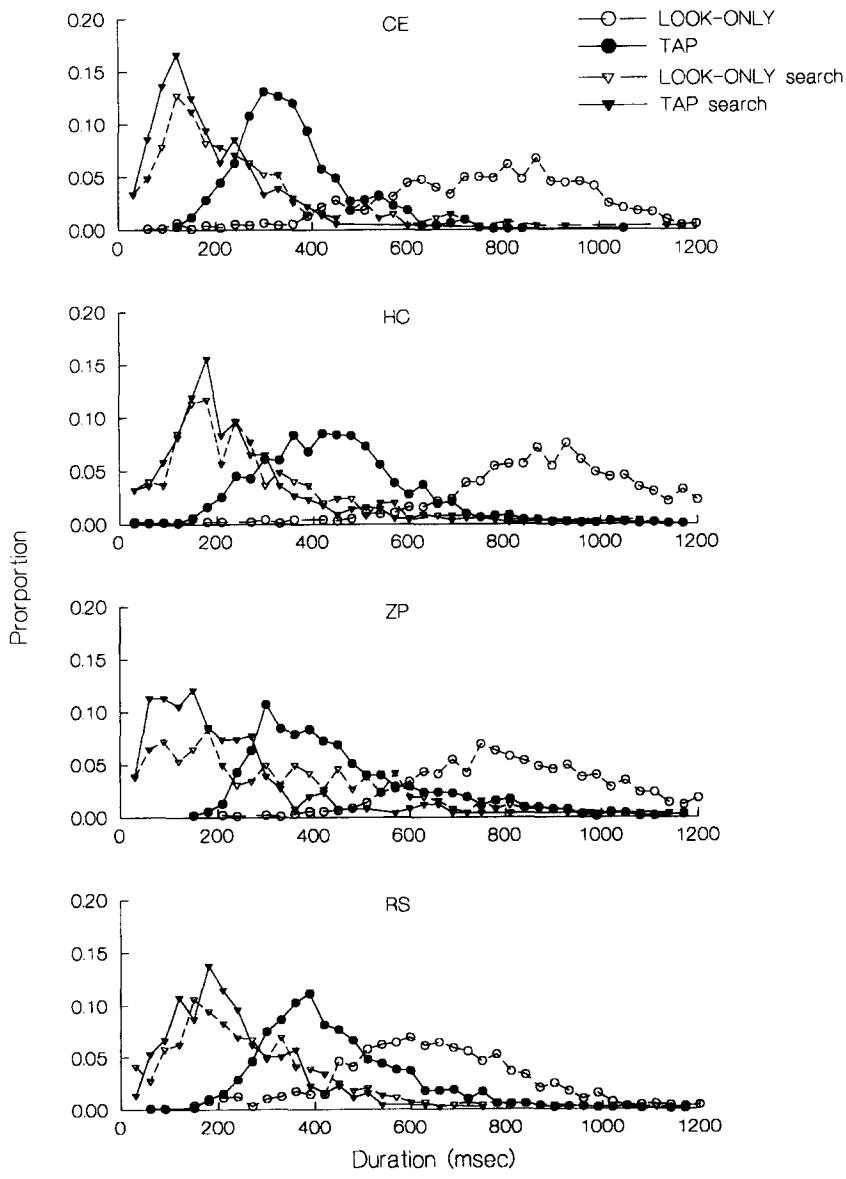


FIGURE 9. Distribution of sequence (circles) and search (triangles) episode durations (msec). Data for tapping is shown with solid lines and solid symbols and data for looking-only are shown with dashed lines and open symbols. Bin width is 30 msec. Data for four subjects are shown in rows labeled CE, HC, ZP, and RS. Each distribution is based on 950–1396 observations for sequence episodes and 247–552 observations for search episodes.

types of episodes have very different duration distributions. Search episode distributions were very similar in both looking and tapping tasks, and they were shorter than sequence episodes. Sequence episodes, on the other hand, were shorter during tapping than during looking-only, a finding consistent with the hypothesis that memory retrieval took more time during looking-only than during tapping.

DISCUSSION

We set out to study global eye movement characteristics in sequential looking and tapping tasks, hoping to be able to use these characteristics to infer the mental operations that underlie their acquisition and eventual skilled performance (see Introduction). We were successful in three major respects: (i) just looking at a sequence of targets and looking at a sequence of targets while tapping them are fundamentally different tasks with memory being more effectively used when looking guides tapping then when looking is undertaken for its own sake; (ii) visual search is a separate task, performed in parallel with looking and tapping; (iii) looking, tapping, visual searching, and remembering are synergistic, they interact in beneficial ways. The third point, how visual, motor, and memorial processes help each other, will be discussed next.

Tapping and looking

The same oculomotor behavior, looking at a sequence of targets, was much faster when it was performed in conjunction with tapping the targets then when it was performed all by itself. This is surprising because it is generally believed that when two independent tasks are performed at the same time, processing resources must be shared, which leads to slower performance on one or both tasks (Sperling & Dosher, 1986; Kowler, Anderson, Dosher & Blaser, 1995). We found that it actually took subjects *less* time to look at targets when they were also required to tap them then when they were only required to look at them. This means that tapping and looking tasks were not independent. Tapping facilitated looking.

This facilitation was not accomplished by a tradeoff between saccadic latency and gaze accuracy because the differences: (i) in the size of the gaze-errors and (ii) in the probabilities and sizes of corrective saccades, were very small between the two tasks (see Fig. 3 and Table 1). This facilitation was also not accomplished by reducing the time spent in visual search because there were not more search episodes when subjects only looked at the targets (see Fig. 7). Furthermore, the extra time did not come from spending more time on visual search, but rather from spending more time looking at each target after gaze had shifted to it (see Fig. 6). Why then, did the subjects dwell longer when they looked then when they tapped? One plausible explanation was that there was more uncertainty as to when the task had been completed. Specifically, when the subject tapped, the subject heard a click every time an element in the sequence was completed successfully, whereas when the subject only looked, no explicit external signal was provided to indicate that it was time to proceed to the next target. The decision to go on to the next target had to be based entirely on the subject's internal criterion for successful completion. This uncertainty hypothesis is clearly plausible, and although this phenomenon may be real, it alone does not explain other aspects of the data, e.g. why sequence episode duration increased over repetitions with the same target configuration (see Fig. 8), or why looking-only benefitted more than tapping from the removal of the target-order constraint. The amount of uncertainty as to when to finish looking at a target remained the same under these conditions. A hypothesis that does explain these aspects of our data, namely that tapping increased memory efficiency, is considered next.

Tapping and memory

The fact that looking-only benefitted more than tapping from the removal of the target-order constraint suggests that memory for target location was working more efficiently when subjects tapped then when they only looked. Memory load was reduced when the subjects selected the target order themselves, because instead of looking for the correct target in the sequence, or searching memory for its location, the subject could simply detect the nearest target in peripheral vision and make a saccade to it. When memory load was reduced in this manner, looking-only became much faster, whereas tapping became faster only with the largest number of targets. Another clue that memory was not working as efficiently during looking-only is the fact that looking-only did not get faster with practice for three of the four subjects, whereas tapping did.

Why should accompanying a gaze-shift with an arm movement to the same location improve the efficiency of memory? There are hints in recent physiological findings. Namely, bimodal, visual–tactile cells were found in the premotor area 6, parietal area 7b, and the putamen (a part of basal ganglia) of monkeys (e.g. Graziano, Yap & Gross, 1994). Receptive fields of these cells mapped nearby space and moved with movements of the arm, or for some cells, with movements of the head or torso. These are the types of cells that would be very active during our tapping task. Perhaps the involvement of dynamic maps of space improved memory for oculomotor commands or target locations. Ours is not the first demonstration that oculomotor performance improves when the hand movements are allowed. Gauthier, Vercher, Ivaldi and Marchetti (1988) have shown that smooth pursuit was faster and pursuit velocity was more accurate when subjects tracked an object with both the finger and the eye than when they tracked it with the eye alone.

Looking and visual search

Ballard *et al.* (1995) have shown that subjects often chose to scan the visual display for information instead of committing this information to memory. One of the subjects in our experiments, RS, did just that. He searched the Worktable, even after 10 repetitions in both tasks. This behavior allowed him to get faster with repetition

when he only looked. The other three subjects, who did not get faster with repetition when they only looked, may have interpreted the instructions more rigorously than RS and tried to reduce the number of "extraneous", out-of-sequence saccades as much as possible as they learned the configuration of targets. In other words, they tried to confine their saccades to gaze-shifts from one to the next target in the sequence. To do this they would have to reduce visual search for targets and rely more on memory of the target's location each time the sequence was repeated. Using memory more in this way caused more time to be spent at each target location because it took longer to access memory for the next target location than to scan the Worktable for it (see Fig. 8). The individual differences observed suggest there is considerable flexibility over whether to use internal memory or visual search, and that subjects may not always chose the best (fastest) strategy available to them.

Looking during tapping

We have evidence, based on experiments done after the initial observations were made, that visual information is needed to tap accurately. We found that subjects could not tap accurately with the eyes closed, even after 10 repetitions with the same configuration. Subjects, sitting with eyes closed, groped around hunting for each target under these conditions. They had only a vague idea about where the target actually was but missed the actual location by wide margins when it was no longer in view. Also, we found that when visual information was reduced by turning off the room-lights and keeping only the targets lighted, subjects spent 50–100 msec more time per target than when the room lights were left on. Reducing visual information slowed tapping down appreciably. Finally, we moved one of the targets over by one grid space on the Worktable after the subject had tapped seven repetitions with the same target configuration, and then required the subject to tap three more repetitions. If the exact target locations or the exact motor commands for the arm to get to the target were remembered, the subject would have spent more time before tapping the relocated target because the initial arm movement would have been to the old target location, and a correction would be required. This did not happen. Moving a target a small, but easily discriminable, distance did not effect the total time to complete a tapping sequence or the time required to tap the relocated target. These observations suggest that even after practice, subjects relied on visual cues associated with fixation, as well as on motor and/or visual memory of the locations of the targets.

Question remaining

The idea that tapping may have been facilitated by memory is consistent both with our data and with the recent physiological findings in monkeys described above. It does not explain, however, why looking was slower than tapping even when target order was selected by the subject. This condition makes minimal demands on memory. Part of the explanation of this puzzling finding may lie in the dynamics of gaze-shifts and in the coordination of the head and the eyes. These possibilities were not considered in this paper. Preliminary analyses have suggested that some portion, but not all, of this difference may be accounted for by characteristics of the gaze-shift dynamics, which are faster in the tapping task than in the looking-only task (Epelboim, Collewijn, Edwards, Erkelens, Kowler, Pizlo & Steinman, 1994c).

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APPENDIX

Computing the Rotation Matrix

Introduction

The rotation matrix, $R(\theta_h,\theta_v,\theta_t)$ is a 3×3 matrix which transforms the vector $\mathbf{x}_{e0}$ (which is the vector from sparker-tip to sighting center expressed as a 3×1 matrix) into the same vector $\mathbf{x}_e$ after the subject's head has moved in such a way so that the location of the sparker-tip is fixed. The full description of how the subject's head has moved will also include a rigid translation but this is accounted for separately as described earlier. The purpose of this Appendix is to describe how the nine elements of the rotation matrix were computed in terms of measured quantities. The complete description consists of two steps. First, the form of the rotation matrix in terms of three angles which are analogous to Fick angles will be derived, and second, the expressions for these angles in terms of the measured head coil angles will be obtained.

Three independent pieces of information are required to uniquely specify a rotation of a vector or coordinate axes: (i) an axis of rotation (defined by a *unit* vector and thus has only two independent components); (ii) the angle through which the vector is rotated about that axis; (iii) the sense of the rotation which specifies which direction of rotation around the axis is positive.

An arbitrary rotation can also be described by a sequence of three rotations around a set of three given axes. In this case three angles are required in order to specify the rotation. This specification will be unique only if the three axes around which the vector is rotated are also given. The description of these three rotations will be termed a procedure. The procedure that was used to specify the above rotation matrix is analogous to the Fick-angle description of eye position.

To obtain the form of $R(\theta_h,\theta_v,\theta_t)$ it is most convenient to determine the passive form of the rotation matrix in which the physical vector remains fixed while the coordinate axes are rotated. The corresponding active rotation matrix, in which the coordinate axes remain fixed while the vector is rotated, is obtained from the passive matrix by taking its inverse. Since all rotation matrices are orthogonal, taking the inverse of the matrix merely involves obtaining its transpose. The rotation procedure described below then becomes a description of how the *coordinate axes* are rotated.

The rotation procedure

The rotation procedure consists of three steps. Step (i) is a rotation of the Worktable coordinate system around the Worktable z-axis through an angle F_1. A new coordinate system is thus obtained which will be referred to as the "primed" system. In step (ii) the primed system is rotated about the primed system's y-axis through an angle F_2 creating another new coordinate system which will be referred to as the "double-primed" system. Step (iii) consists of rotating the double-primed system about its x-axis through an angle F_3 creating the final coordinate system.

Each step in the sequence of three rotations can be represented by a simple rotation matrix since each rotation is about one of the coordinate axes of the system. Thus the full rotation matrix is written as

$$R(F_1,F_2,F_2) = [R_{x''}(F_3)R_{y'}(F_2)R_z(F_1)]^T. \tag{A1}$$

Where the individual matrices have the form

$$R_z(F_1) = \begin{pmatrix} \cos(F_1) & \sin(F_1) & 0 \\ -\sin(F_1) & \cos(F_1) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{A2}$$

$$R_{y'}(F_2) = \begin{pmatrix} \cos(F_2) & 0 & \sin(F_2) \\ 0 & 1 & 0 \\ -\sin(F_2) & 0 & \cos(F_2) \end{pmatrix}, \tag{A3}$$

$$R_{x''} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(F_3) & -\sin(F_3) \\ 0 & \sin(F_3) & \cos(F_3) \end{pmatrix}. \tag{A4}$$

Inserting these three matrices into equation (A1) gives the form of the full rotation matrix expressed in terms of the angles (F_1,F_2,F_3)

$$R(F_1,F_2,F_3) = \begin{pmatrix} c_1c_2 & -s_1c_3 + c_1s_2s_3 & -s_1s_3 - c_1s_2c_3 \\ s_1c_2 & c_1c_3 + s_1s_2s_3 & c_1s_3 - s_1s_2c_3 \\ s_2 & -c_2s_3 & c_2c_3 \end{pmatrix}, \tag{A5}$$

where $s_i = \sin(F_i)$, $c_i = \cos(F_i)$, and $i = 1, 2, 3$. This matrix represents the performance of the Fick rotation procedure on the vector from the sparker-tip to the sighting center. It will not change even if the exact nature of the apparatus changes. What would change are the expressions of the angles (F_1,F_2,F_3) in terms of the measured angles $(\theta_h,\theta_v,\theta_t)$. These expressions, valid for the MRFM apparatus, are derived below.

Expressing (F_1,F_2,F_3) in terms of $(\theta_h,\theta_v,\theta_t)$

The derivation of the expressions of the Fick-like angles (F_1,F_2,F_3) in terms of the measured head coil angles $(\theta_h,\theta_v,\theta_t)$ is based directly on the definitions of these latter angles. The basic procedure will be to define the form of the unit vectors normal to the head coils in the standard configuration (see Fig. A1) and then to perform the Fick rotation procedure on these unit vectors and finally to apply the definitions of the head-angles to the rotated unit vectors.

The horizontal, vertical, and torsional head angles and their relationships with the worktable coordinates are illustrated in Fig. A2 The horizontal angle, θ_h [Fig. A2(a)], is the angle between the orthographic projection of the unit vector normal to the large head coil onto the Worktable xy-plane and the worktable x-axis. The vertical angle, θ_v [Fig. A2(b)], is the angle between the orthographic projection

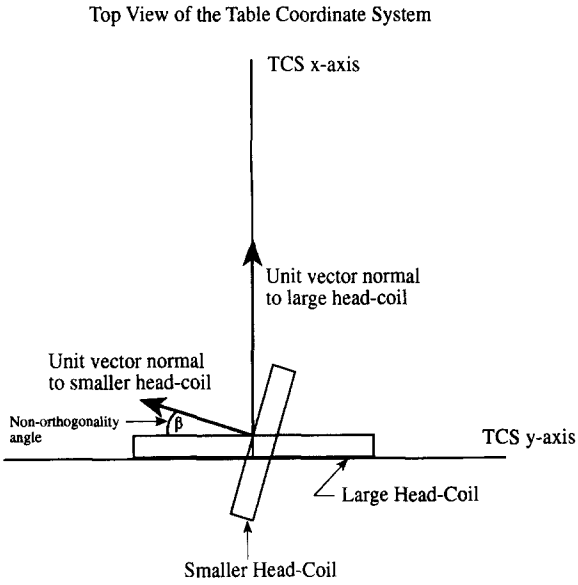

FIGURE A1. The top view of the Worktable coordinate system and the head-coli apparatus. Non-othogonality angle β is exaggerated for clarity. See text for explanation.

of this same unit vector onto the Worktable xz-plane and the worktable x-axis. The torsion angle, θ_t [Fig. A2(c)], is the angle between the orthographic projection of the unit vector normal to the smaller head coil onto the Worktable yz-plane and the worktable y-axis. Mathematically, if the components of these unit vectors are known, the orthographic projection onto the xy-plane is performed by setting the z-component of the unit vector to zero; the orthographic projection to the xz-plane is performed by setting the y-component to zero; finally, the orthographic projection onto the yz-plane, is performed by setting the x-component to zero.

Figure A1 shows the top view of the head coils when in standard configuration. Note that the large and smaller head coil are not precisely perpendicular. The angle representing this departure from othogonality will be denoted by β ($\beta = 48.3$ min arc). In this configuration, the unit vectors normal to the coils both lie in the xy-plane of the Worktable coordinate system and the unit vector normal to the large head coil points along the Worktable x-axis. The unit vector normal to the smaller head coil lies along a ray in the xy-plane which is inclined at the angle β with respect to the Worktable negative y-axis. The procedure for expressing (F_1,F_2,F_3) in terms of $(\theta_h,\theta_v,\theta_t)$ will be illustrated carefully for F_1 and the results for the other two angle will merely be stated.

The basic procedure for determining $F_1(\theta_h,\theta_v,\theta_t)$ consists of writing Worktable coordinates of applying the rotation matrix $R(F_1,F_2,F_3)$ to the large head coil unit normal vector in the standard configuration to obtain this vector's components in an arbitrary orientation described by angles (F_1,F_2,F_3). Afterwards, the orthographic projection onto the Worktable xy-plane is performed and, finally, the definition of θ_h is used to get a relationship between it and F_1.

From Fig. A1 it can be seen that, in the standard configuration, the unit vector normal to the large head coil points along the Worktable x-axis. Thus, this vector is represented as the following column matrix

$$\mathbf{u}_{\mathbf{std}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \tag{A6}$$

Left-multiplication of this column matrix by $R(F_1, F_2, F_3)$ given in Equation (A5) yields

$$\mathbf{u}_{\mathbf{arb}} = \begin{pmatrix} c_1c_2 \\ s_1c_2 \\ s_2 \end{pmatrix}. \tag{A7}$$

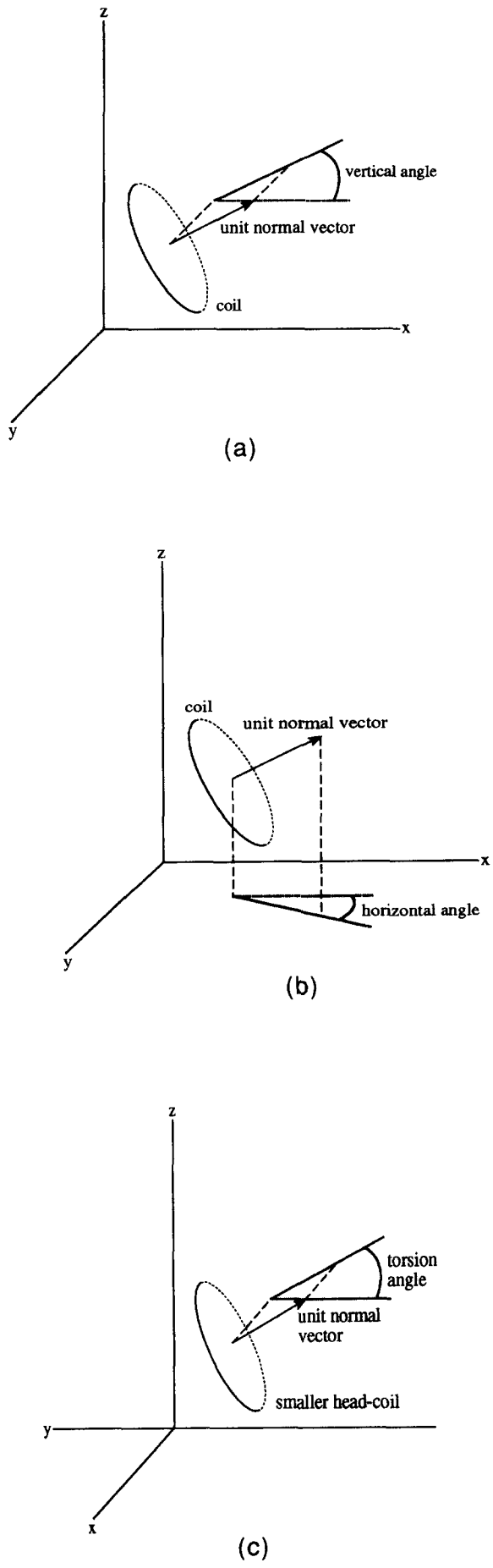

FIGURE A2. Definitions of the horizontal (a), vertical (b), and torsion (c) head-angles with respect to the Worktable coordinate system. See text for explanation.

Orthographic projection of this vector onto the Worktable xy-plane is obtained by setting the z-component to zero. The angle between the resulting vector and the Worktable x-axis is the measured angle, θ_h. Thus

$$\tan(\theta_h) = \frac{(\mathbf{u}_{arb})_y}{(\mathbf{u}_{arb})_x} = \tan(F_1). \tag{A8}$$

So the final relationship becomes simply

$$F_1 = \theta_h. \tag{A9}$$

Following a similar procedure, expressions for $\sin(F_2)$, $\cos(F_2)$, $\sin(F_3)$, and $\cos(F_3)$ are found. The results are

$$\sin(F_2) = \frac{\cos(\theta_h)\tan(\theta_v)}{(1 + \cos^2(\theta_h)\tan^2(\theta_v))^{1/2}}, \tag{A10}$$

$$\cos(F_2) = \frac{1}{(1 + \cos^2(\theta_h)\tan^2(\theta_v))^{1/2}}, \tag{A11}$$

$$\sin(F_3) = \frac{t_3}{(1 + t_3^2)^{1/2}}, \tag{A12}$$

and

$$\cos(F_3) = \frac{1}{(1 + t_3^2)^{1/2}}. \tag{A13}$$

Where

$$t_3 = \frac{-ab - c(a^2 + b^2 - c^2)^{1/2}}{a^2 - c^2}, \tag{A14}$$

with

$$a = s_1 s_2 t_t - c_2, \quad b = c_1 t_t, \quad c = (s_2 + s_1 c_2 t_t)^{1/2} t_\beta, \tag{A15}$$

and

$$t_t = \tan(\theta_t), \quad t_\beta = \tan(\beta). \tag{A16}$$