# *Human interaction in solving hard practical optimization problems*

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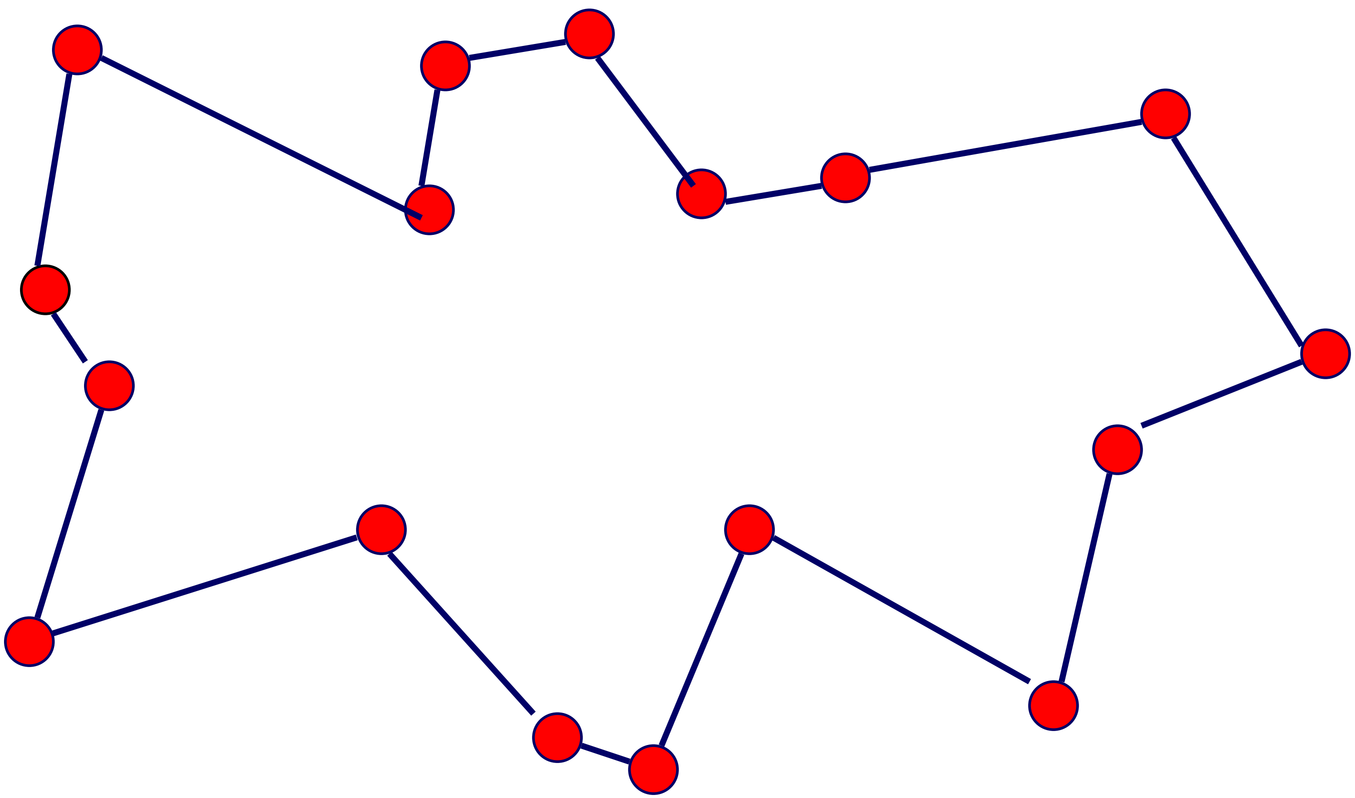
# Outline

- Optimization problems in Operational Research
- Winter Gritting case study
- Reasons for using human interaction in practical optimization problems
- Issues and questions

# Introduction

- OR often involves solving a constrained optimization problem
- Sometimes this is easy, e.g. Linear Programming for continuous variables
- Sometimes this is hard, e.g. large scale NP-Hard problems
- All known algorithms to solve NP-Hard problems to proven optimality have a running time that is exponential in the size of the problem. (e.g. the TSP)

# Travelling Salesman Problem

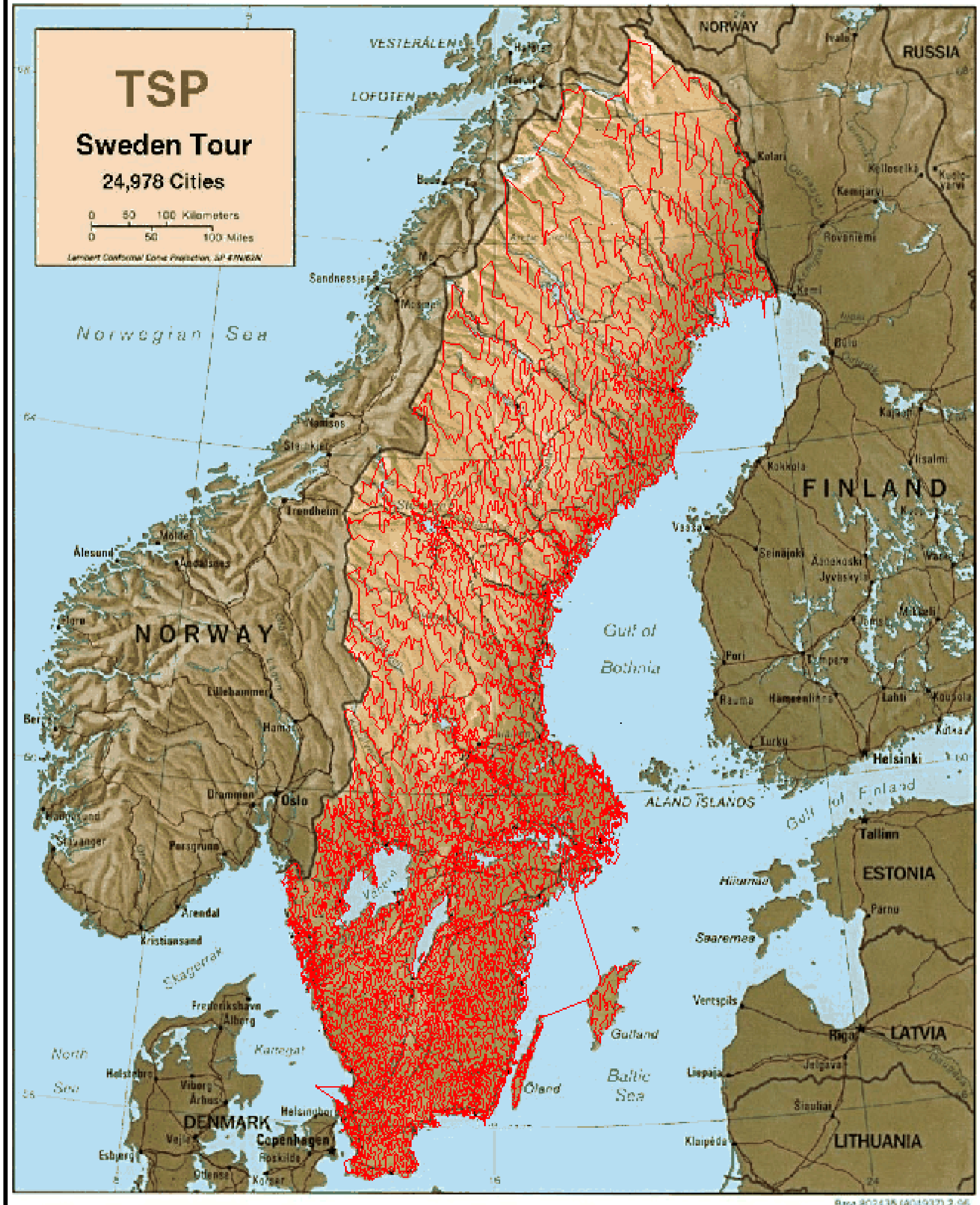


# Travelling Salesman Problem

- Find a minimum cost circuit visiting all the vertices of a graph
  - Problem is NP-Hard
  - Many heuristics have been developed
  - Exact methods can solve large problems to optimality
- Recent exact results on the TSP can be found on http://www.tsp.gatech.edu/

# The TSP Record

In May 2004, the traveling salesman problem of visiting all 24,978 cities in Sweden was solved: a tour of length 855,597 TSPLIB units (approximately 72,500 kilometers) was found and it was proven that no shorter tour exists. This is currently the largest solved TSP instance, surpassing the previous record of 15,112 cities through Germany set in April 2001.


TSP
Sweden Tour
24,978 Cities
0 50 100 Kilometers
0 50 100 Miles
Lambert Conformal Conic Projection, SP 47N/62N
Norwegian Sea
NORWAY
FINLAND
RUSSIA
ESTONIA
LATVIA
LITHUANIA
DENMARK
Gulf of Bothnia
Gulf of Finland
Baltic Sea
North Sea
ALAND ISLANDS
Gotland
Öland
Saaremaa
Hiiumaa
Skagerrak
Kattegat
VESTERÅLEN
LOFOTEN
Oslo
Helsinki
Tallinn
Riga
Copenhagen
Trondheim
Ålesund
Molde
Andalsnes
Lillehammer
Drammen
Porsgrunn
Arendal
Kristiansand
Stavanger
Haugesund
Bergen
Bodø
Sandnessjøen
Mosjøen
Namsos
Steinkjer
Harstad
Narvik
Kolari
Kemijarvi
Rovaniemi
Kelloselkä
Kemi
Oulu
Kajaani
Kokkola
Vaasa
Seinäjoki
Jyväskylä
Tampere
Pori
Rauma
Hämeenlinna
Lahti
Kouvola
Kotka
Turku
Pärnu
Ventspils
Liepāja
Jelgava
Šiauliai
Klaipėda
Frederikshavn
Ålborg
Viborg
Århus
Holstebro
Esbjerg
Vejle
Odense
Korsør
Roskilde
Helsingborg
Base 802438 (A04937) 2-96

# *Computational effort*

- The majority of the work was carried out on a cluster of 96 dual processor Intel Xeon 2.8 GHz workstations running as a background process when the machines were not otherwise active.
- The cumulative CPU time used in the five branch-and-cut runs and in the cutting-plane procedures for the five root LPs was approximately 84.8 CPU years on a single Intel Xeon 2.8 GHz processor

# Travelling Salesman Problem

- Improvements in performance are due to:
  - Faster computers
  - Improved theory
  - Good computer programming

# Practical Vehicle Routing Problems

- Almost always have additional features
- E.g. Capacity constraints, time windows, different commodities with different requirements etc.
- These features tend to make the problems harder
- So heuristic methods are needed to give good quality solutions in a reasonable computing time.
- Should the heuristic be automatic or involve human interaction?

# *Case study – Winter Gritting*

- Winter gritting is a type of arc routing problem.
- Arc routing problems arise in practice when roads require treatment or customers must be served who are located along roads.
- M. Dror, editor (2000) *Arc Routing: Theory Solutions and Applications*. Kluwer, Boston. ISBN 0-7923-7898-9

# Practical examples include

- Postal deliveries
- Parking meter collection
- Meter reading
- Refuse collection
- Snow ploughing
- Road sweeping
- Winter gritting

# *Winter gritting*

# Winter Gritting

The firstday of Spring - 8 inches of snow and driving conditions are difficulton the Cheshire/Derbyshire border.

# Chinese Postman Problem

- If all arcs in a network require to be covered on a tour and there are no other constraints, the problem of finding the minimum length tour is called the Chinese Postman Problem (CPP).
- Meigu Guan (Kwan Mei-Ko) Chinese Mathematics 1962
- The CPP is not NP-Hard; there is an efficient algorithm that will find the optimal solution.
- However the addition of constraints on capacity and/or time make the problem NP-Hard.

# Winter gritting constraints

- Vehicles have limited salt capacity; at normal rates of spreading this implies a constraint on the length of road that can be treated before the gritter must be refilled at the depot or a location storing salt.
- Roads are divided into categories with different time deadlines for treatment. Typically, the highest priority roads must be treated within 2 hours.
- Roads may be one-way or two-way.

# The Time Constraint Two Phases Algorithm

- Constructive heuristic
  - Phase 1 adds roads from near the boundary of the region back to the start at the depot
  - Phase 2 extends the route by adding roads to the current end of the route
  - Route choices based on a simple set of priority rules
  - Resulting routes are always feasible with respect to the constraints

# *Allowing human interaction*

- Build up of routes represented on a map
- Colour of roads indicates progress
- Information also given on gritting distance and time spent so far
- At each step, user can accept the proposed next road to be added or…
- Override and include a different road or…
- Refill gritter with salt or…
- Deadhead elsewhere

# *Results of tests in Lancashire*

| **Area** | **No. Nodes** | **No. Arcs** | **Method** | **No. Vehicles** | **Distance (km)** |
|---|---|---|---|---|---|
| **East** | 77 | 111 | TCTPA | 7 | 366 |
| | | | User-intervened | 7 | 339 |
| **South** | 140 | 203 | TCTPA | 12 | 640 |
| | | | User-intervened | 10 | 532 |
| **North** | 254 | 380 | TCTPA | 20 | 1124 |
| | | | User-intervened | 17 | 927 |

# *Reasons for using human interaction*

- Strong visual representation that allows human judgement to guide the solution
- Development time available
- Make use of human experience or expertise (e.g. forecasting)
- Modifying details not covered by automatic method (e.g. aircraft loading, VRP solutions)
- Learning and training
- Less well-defined problems (e.g. relaxing soft constraints or not)

# Issues for further research

- Visualization and optimization
  - What are the best visualizations for hard optimization problems?
  - E.g. 2D v Virtual reality, animation of algorithms
- Interaction and optimization
  - What are the best ways to make interaction easy and effective?
  - Is there a way to measure the trade-off between an interactive approach and a more complex automatic heuristic?