Purdue Workshop,
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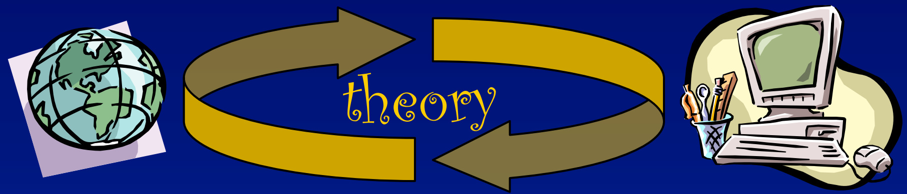

Understanding Human Optimization: The Case for a Tractable-design Cycle

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Overview

Formulating cognitive theories

- Levels of Explanation
- Optimization and Satisficing

Testing cognitive theories

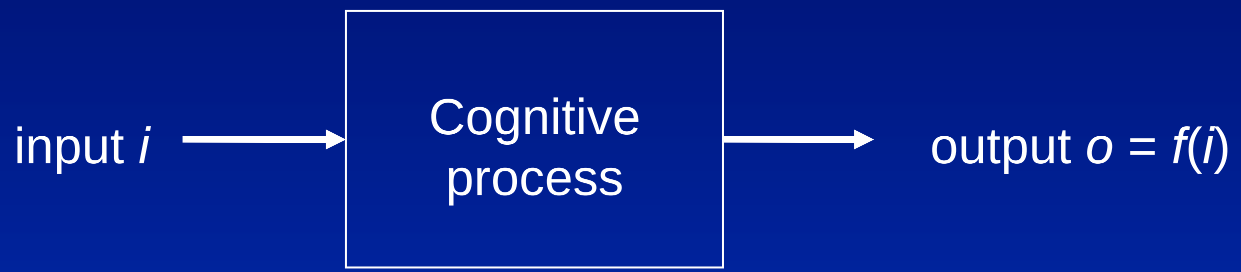
- Underdetermination of Theory
- Utilizing Theoretical Constraints

Tractable-design cycle

- Cautions and Clarifications

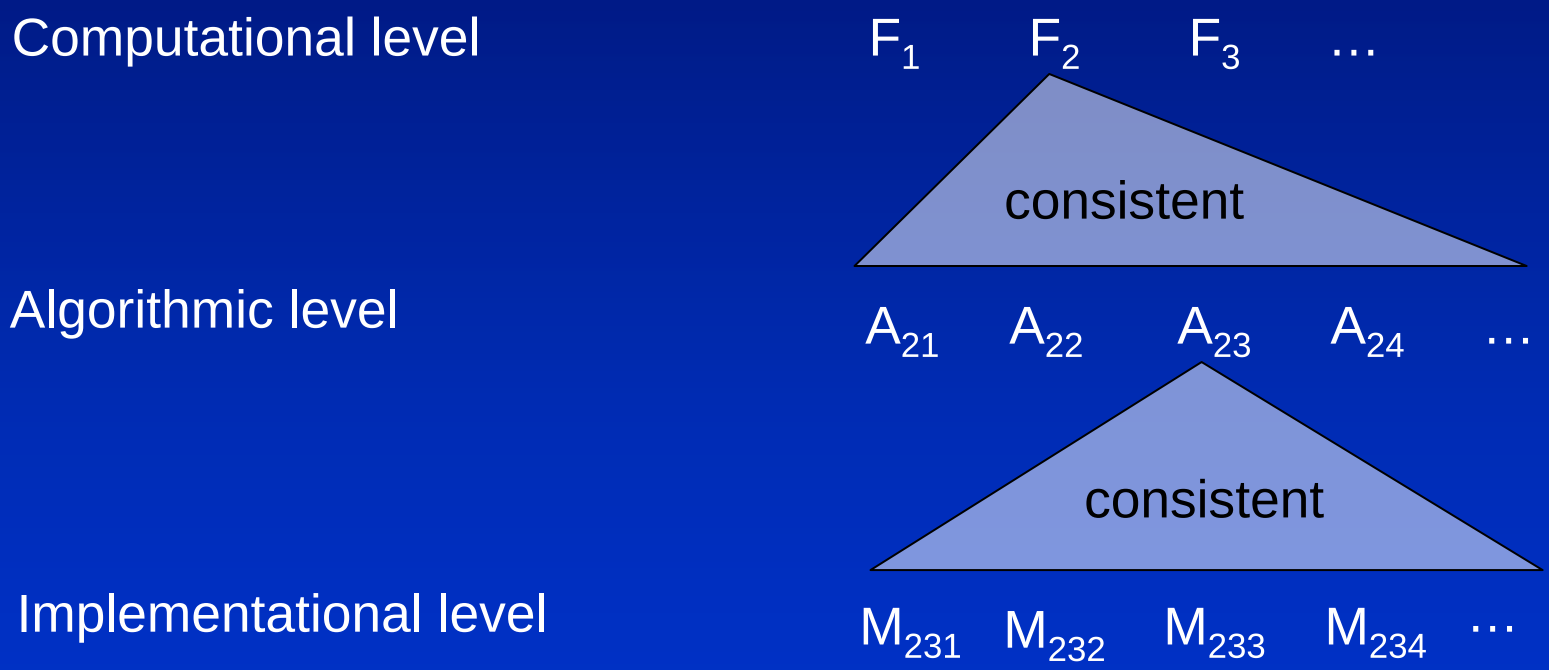
Conclusion

Levels of Description



Level	Marr's levels	Question
1	Computational	What?
2	Algorithm	How_1?
3	Implementation	How_2?

Underdetermination of Lower Levels



Domain	Computational Level Theory (Informal)	References
Categorization	Input: A set of objects. Output: A partition that **maximizes** within-category similarity and between-category dissimilarity.	(Pothos & Chater, 2001, 2002; Rosch, 1973)
Coherence	Input: A set of interconnected beliefs. Output: A truth assignment of **maximum** coherence.	(Millgram, 2000; Thagard, 2000; van Rooij, 2003)
Perceptual organization	Input: A set of visual elements. Output: A grouping of **maximum** simplicity.	(van der Helm & Leeuwenberg, 1996; van der Helm, 2004)
Similarity	Input: Two objects, x and y. Output: The length of the **shortest** program computing x from y.	(Hahn, Chater, & Richardson, 2003; Chater and Vitanyi, 2003)
Subset Choice	Input: A set of alternatives. Output: A subset of **maximum** value.	(Fishburn & LaValle, 1993, 1996; van Rooij, Stege & Kadlec, 2005)
Visual matching	Input: A target, display and criteria x and y. Question: Do target and display match on **at least** x aspects and mismatch on **at most** y aspects?	(Kube, 1990, 1991; Tsotsos, 1990, 1991; van Rooij, 2003).

Formalization Example 1

Categorization (informal)

Input: A set of objects.

Output: A partition that maximizes within-category similarity and between-category dissimilarity.

Categorization (formal)

Input: A set of objects, A, with a similarity measure $s(x,y)$ and a dissimilarity measure $d(x,y)$ for each pair of objects $x, y \in A$.

Output: A partition of A into categories $A_1, A_2, \ldots, A_k$, such that

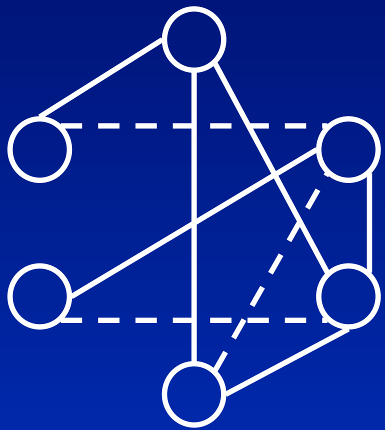
$$\sum_{x,y \in A_i, i=1,2,\ldots k} s(x,y) + \sum_{x,y \notin A_i, i=1,2,\ldots k} d(x,y)$$ is maximum.

Formalization Example 2

Coherence (informal)

Input: A set of interconnected beliefs.

Output: A truth assignment of maximum coherence.



Coherence (formal)

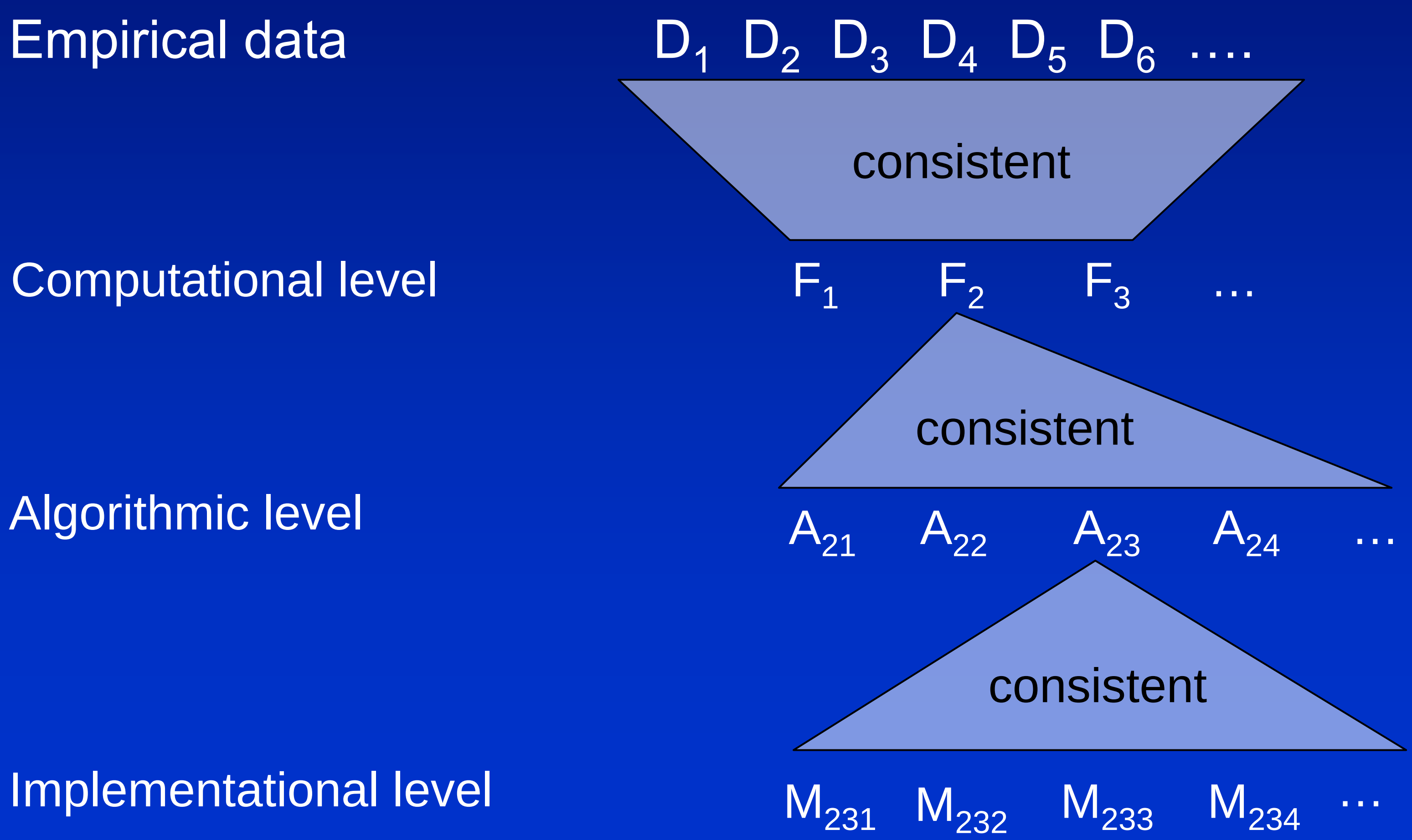
Input: Set of propositions P, set of constraints $C = C^{-} \cup C^{+}$.

Output: A truth assignment to the propositions in P that satisfies a maximum number of constraints.

A constraint $(p, q) \in C^{-}$ is satisfied if p is 'false' and q is 'true'.

A constraint $(p, q) \in C^{+}$ is satisfied if both p and q are 'true' or both p and q are 'false'

Empirical Underdetermination of the Computational Level

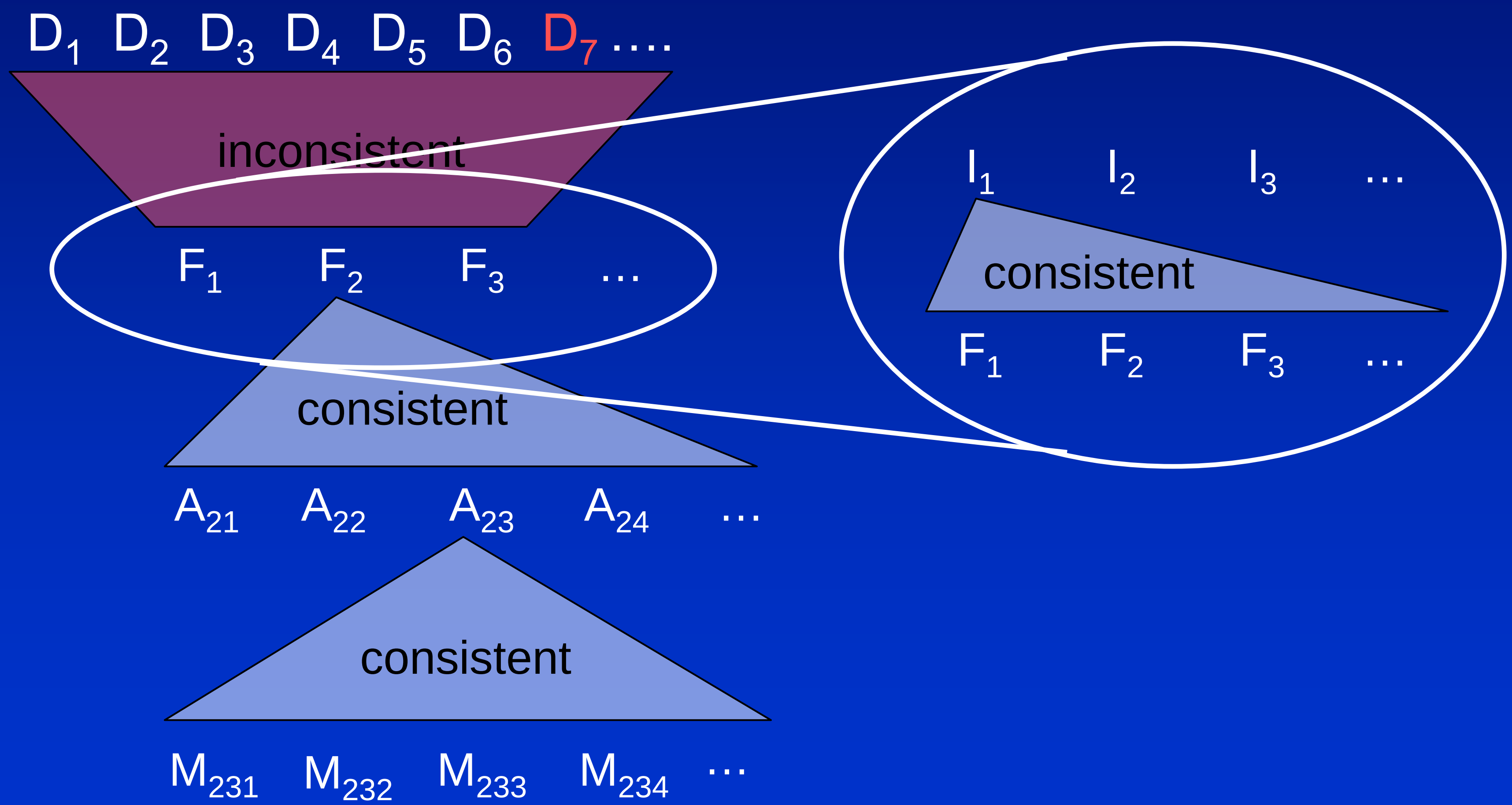


Empirical Underdetermination of the Computational Level

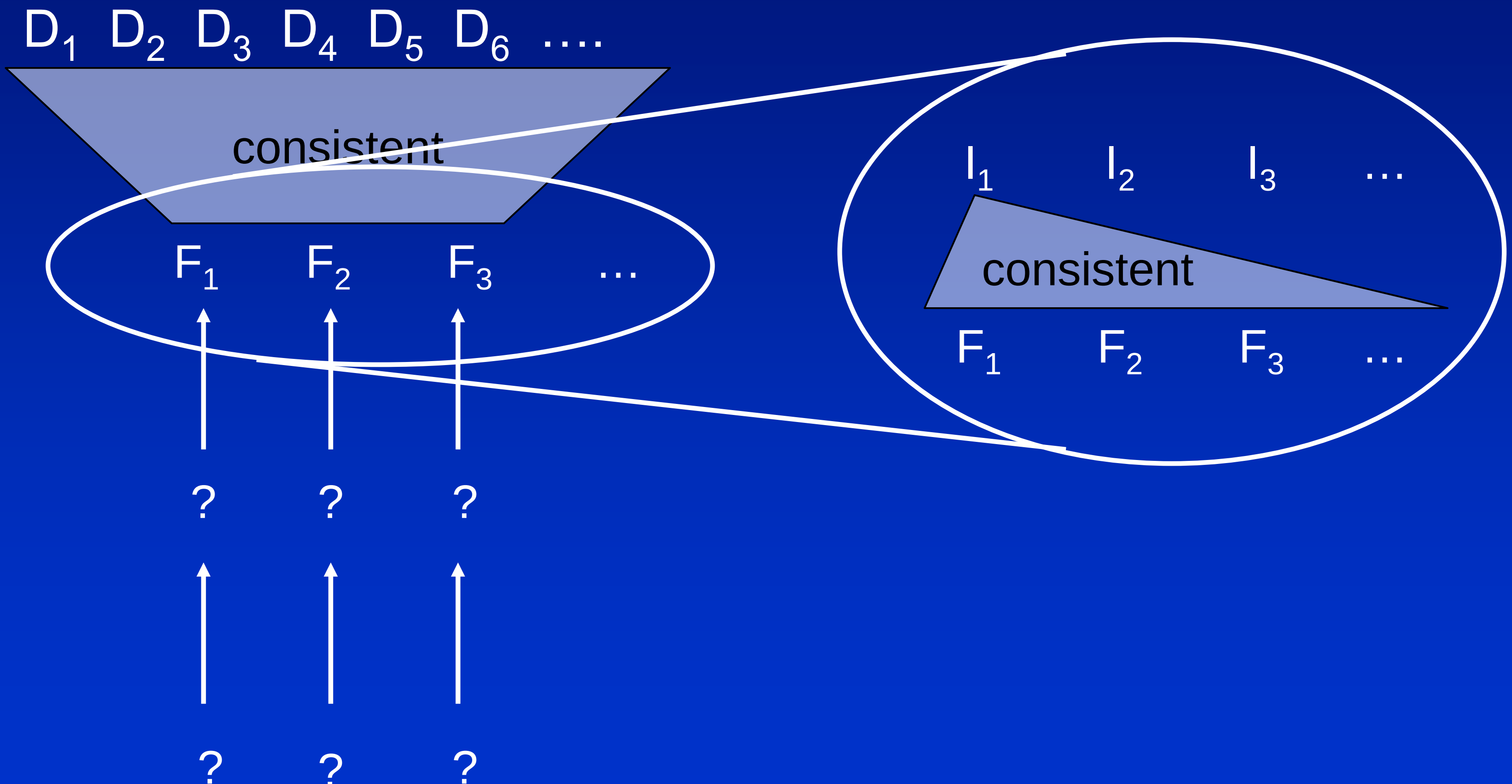
Several reasons

1. Any finite set of input-output observations is consistent with infinitely many different functions.
2. Inputs and outputs are usually not directly observable.
3. Psychological data are noisy (due to context variables not under the control of the experimenter).
4. Commitment is usually to the informal theory, not the formalization.

Even More Underdetermination …



Can We Use Lower-Level Constraints?



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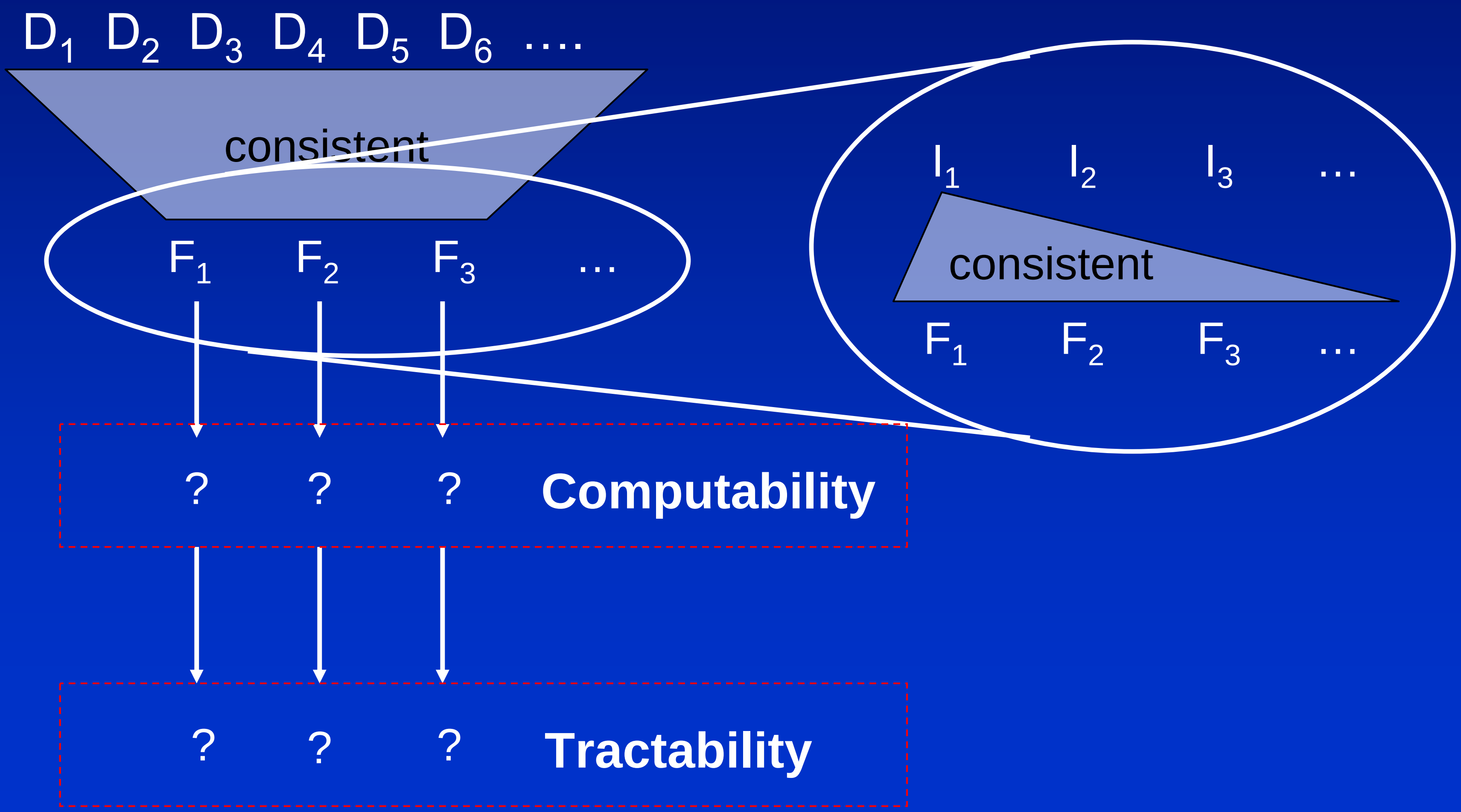
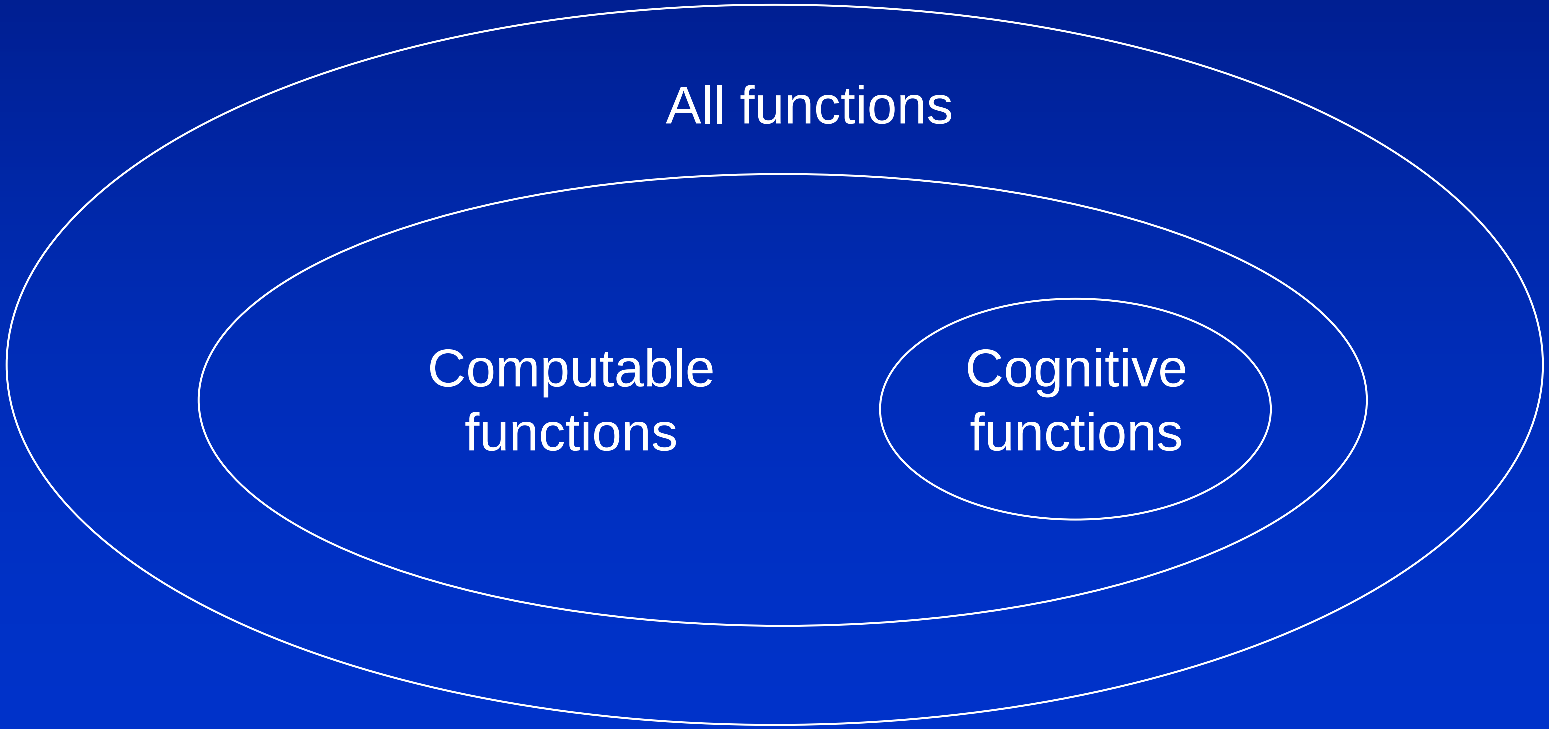


Computability Constraint

Cognitive functions $\subseteq$ Computable functions



Tractability Constraint

Observation 1:

Cognitive functions are implemented by <u>physical</u> systems.

Observation 2:

Physical systems are <u>limited</u> in space and speed.

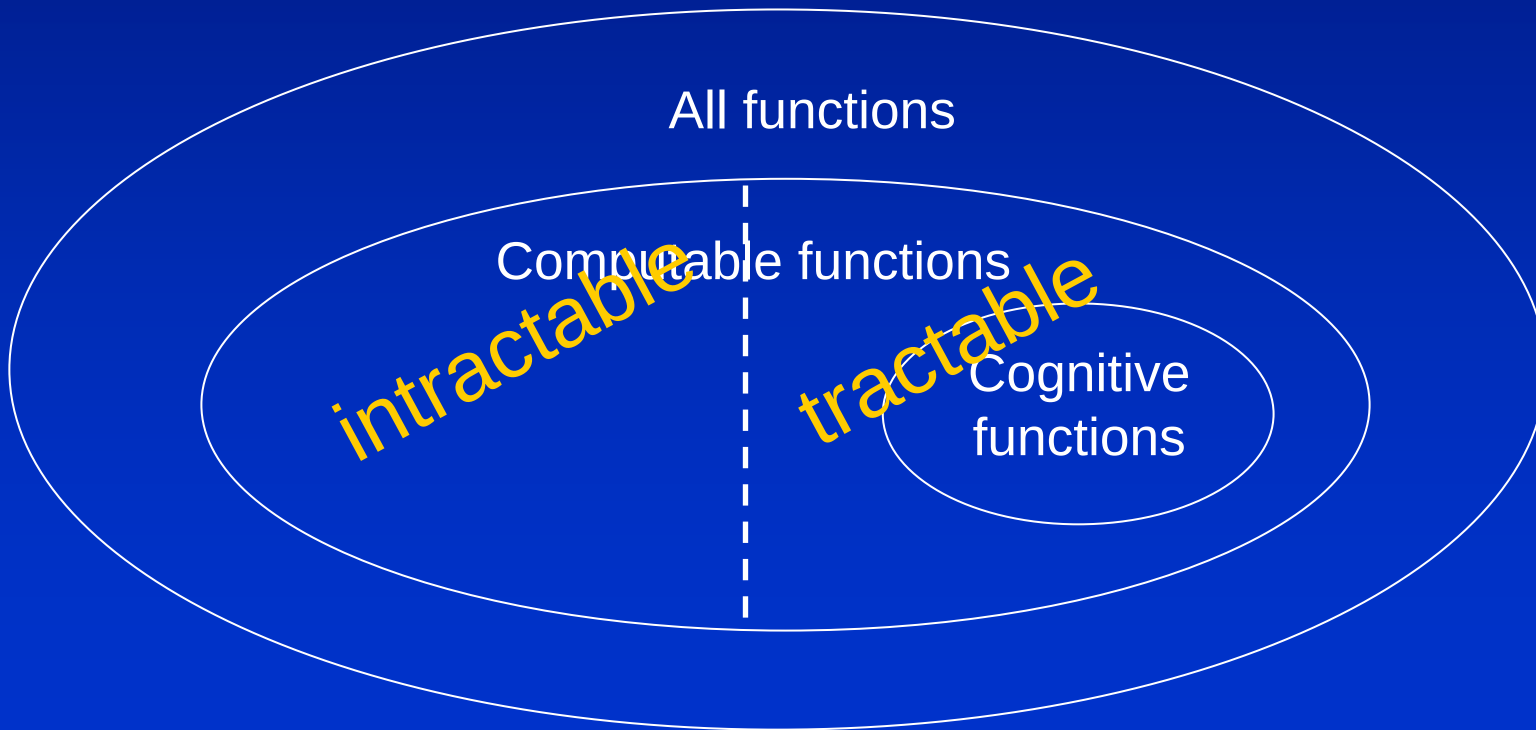
Conclusion:

Cognitive functions $\subseteq$ Tractable functions.

[e.g. Frixione, 2001; Simon, 1990; Thagard & Verbeurgt, 1998]

Tractability Constraint

Cognitive functions $\subseteq$ Tractable functions



Is Rosch's Categorization Tractable?

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Input: A set of objects.

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$$\sum_{x,y \in A_i, i=1,2,\ldots k} s(x,y) + \sum_{x,y \notin A_i, i=1,2,\ldots k} d(x,y)$$

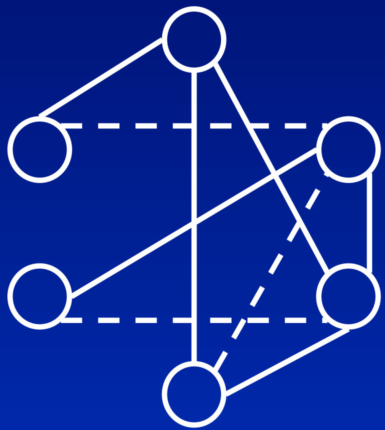
is maximum.

Is Thagard's Coherence Tractable?

Coherence (informal)

Input: A set of interconnected beliefs.

Output: A truth assignment of maximum coherence.



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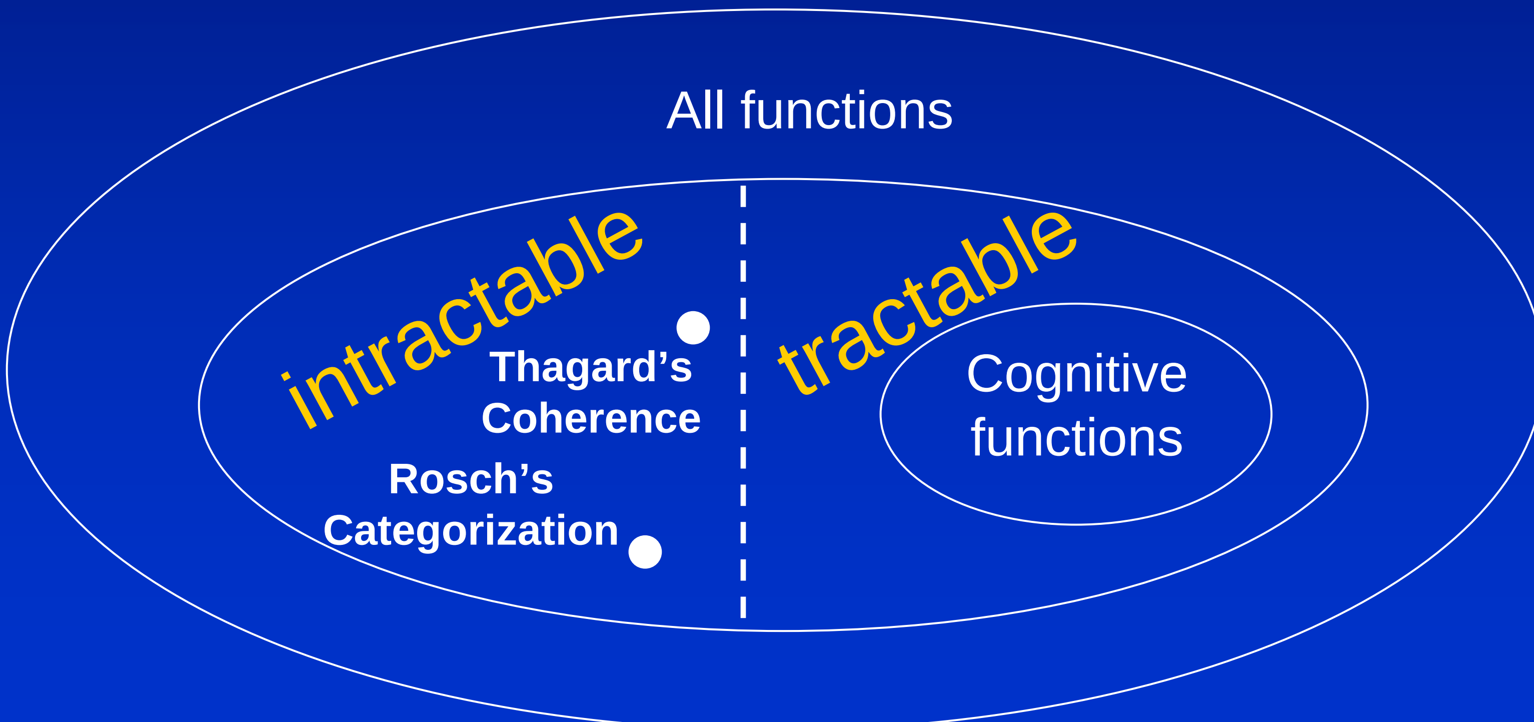
Unbounded Exponential-time Computation is Intractable

Exhaustive search of combinatorial complex spaces is impractical for all but very small input sizes.

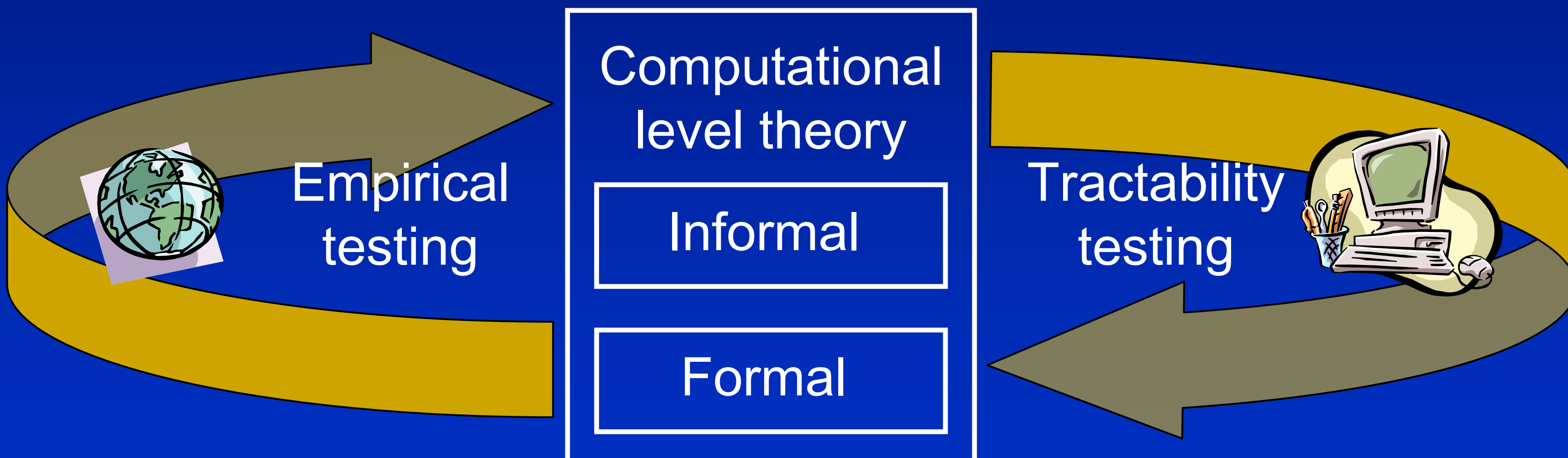
n	$O(n^2)$	$O(2^n)$
5	0.15 msec	0.19 msec
20	0.04 sec	1.75 min
50	0.25 sec	8.4×10^{2} yrs
100	1.00 sec	9.4×10^{17} yrs
1000	1.67 min	7.9×10^{288} yrs

Rosch's Categorization and Thagard's Coherence are NP-hard

Cognitive functions $\subseteq$ Tractable functions



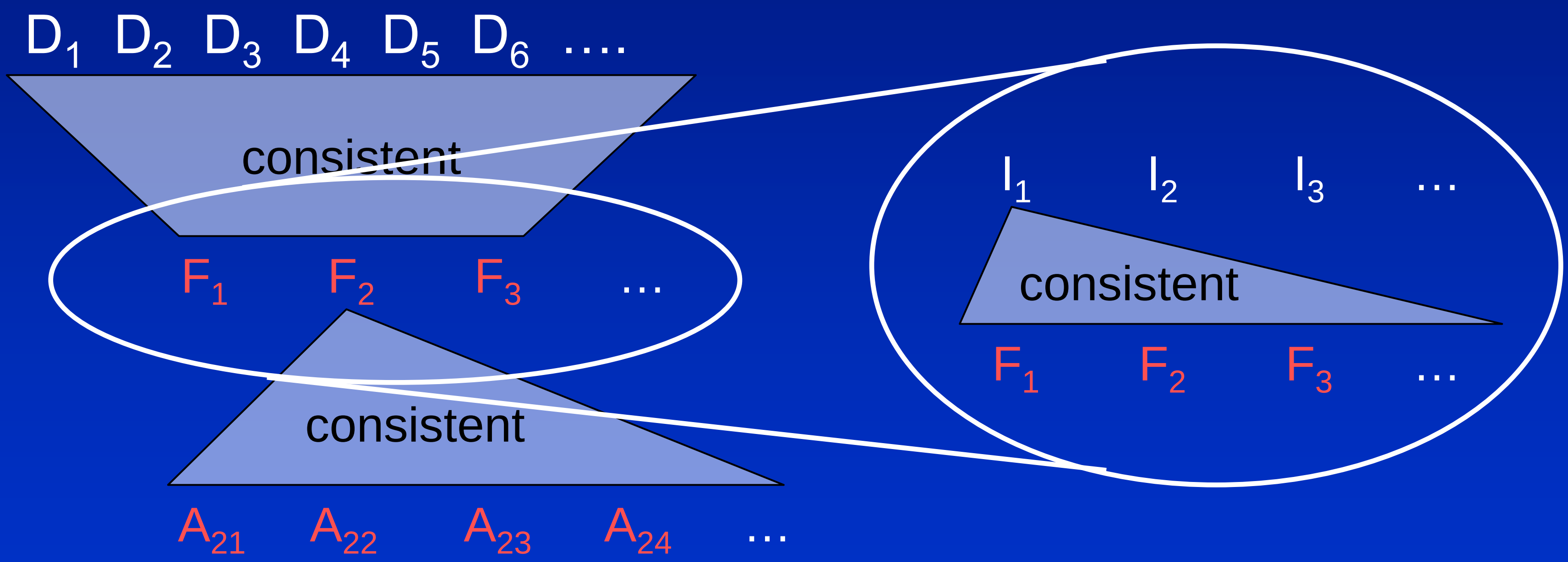
Empirical Cycle + Tractable-design Cycle



Cautions and Clarifications

Intractability is not always bad news!

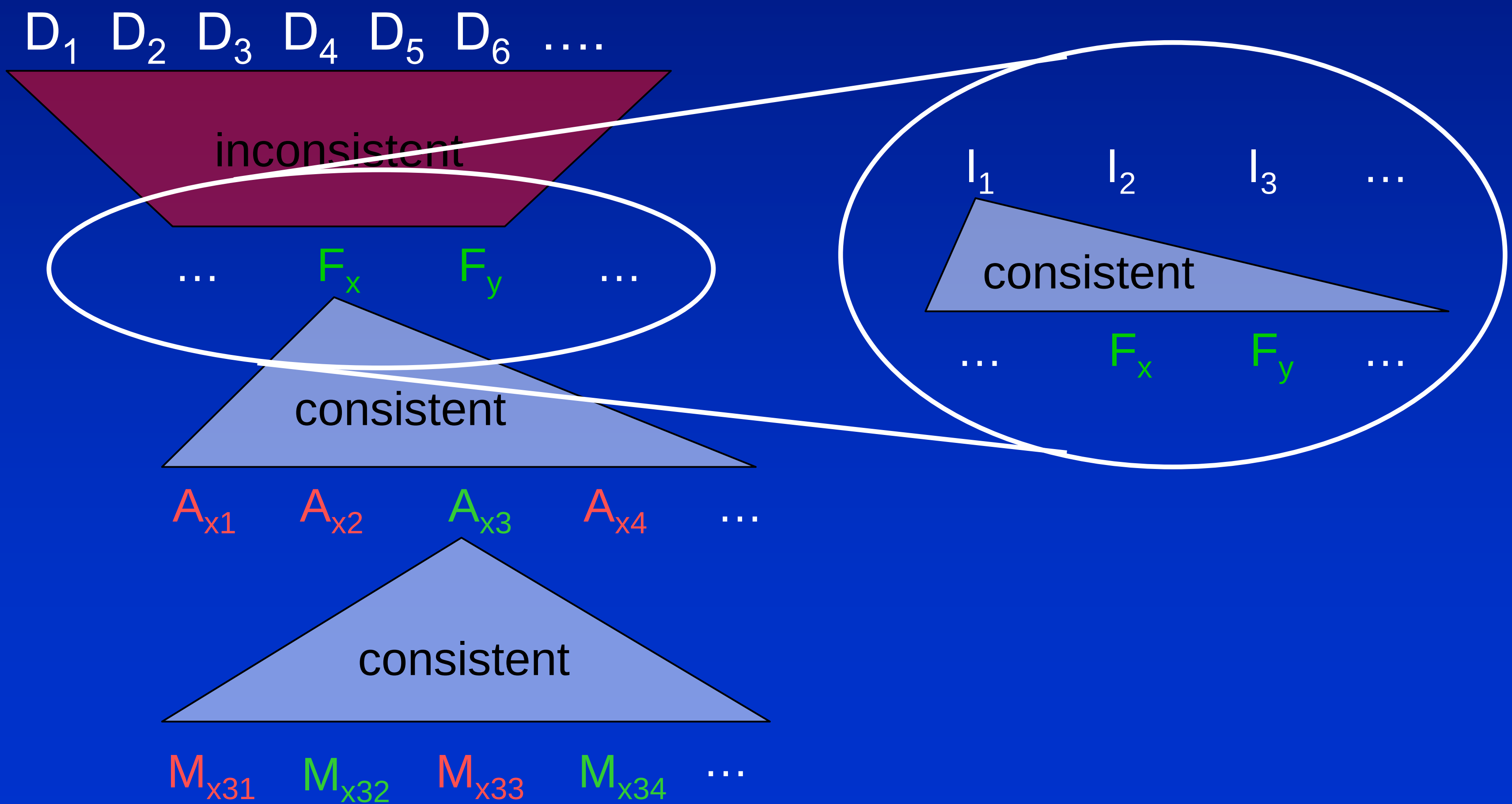
(Or, at least: don't shoot the messenger)



Cautions and Clarifications

Tractability is not always good news!

(Or, at least: it is not a goal in itself)



Cautions and Clarifications

Tractability is not trivially achieved!

For example:

Optimization is tractable $\Leftrightarrow$ Satisficing is tractable

Coherence (optimization variant)

Input: Set of propositions P, set of constraints $C = C^{-} \cup C^{+}$.

Output: A truth assignment to the propositions in P that satisfies a **maximum** number of constraints.

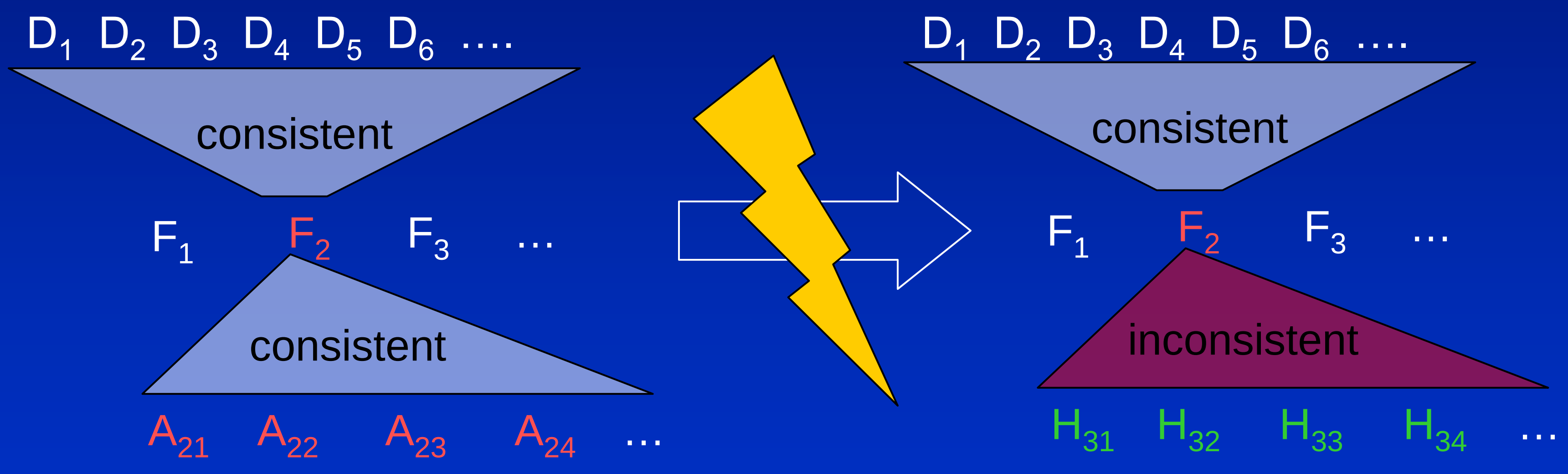
Coherence (satisficing variant)

Input: Set of propositions P, set of constraints $C = C^{-} \cup C^{+}$, integer k.

Output: A truth assignment to the propositions in P that satisfies **at least *k*** constraints.

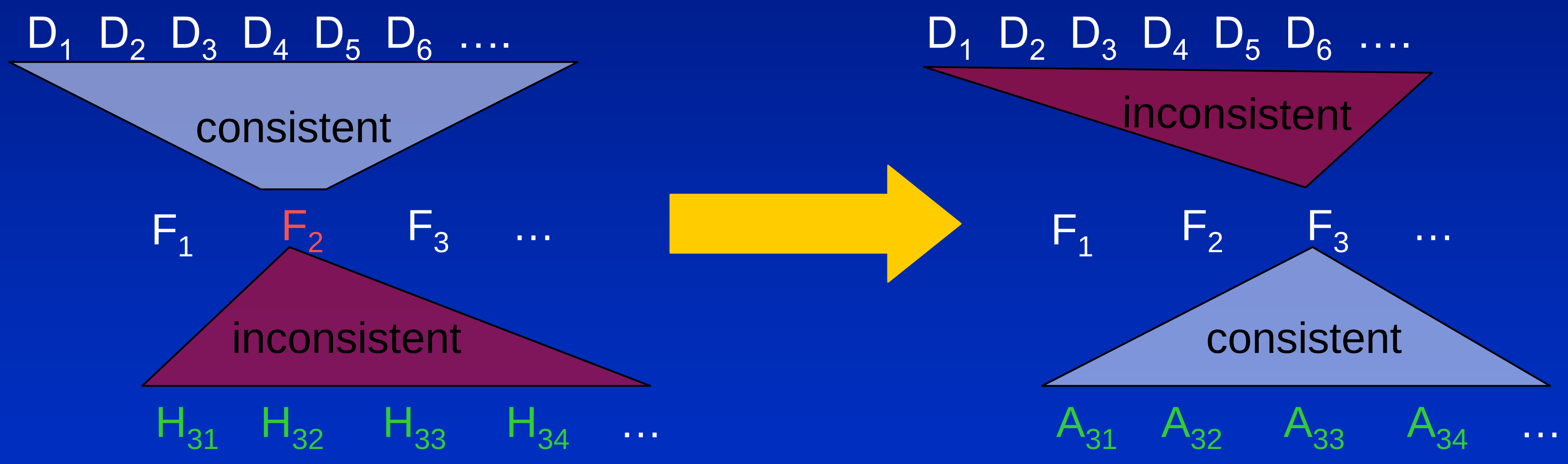
Cautions and Clarifications

Heuristics cannot serve as algorithmic level theories!



Cautions and Clarifications

Intractability requires theory change!



Cautions and Clarifications

Domain restriction is a form of theory change!

For example:

Coherence (unrestricted)

Input: Set of propositions P, set of constraints $C = C^{-} \cup C^{+}$.

Output: A truth assignment to the propositions in P that satisfies a **maximum** number of constraints.

Coherence (restricted)

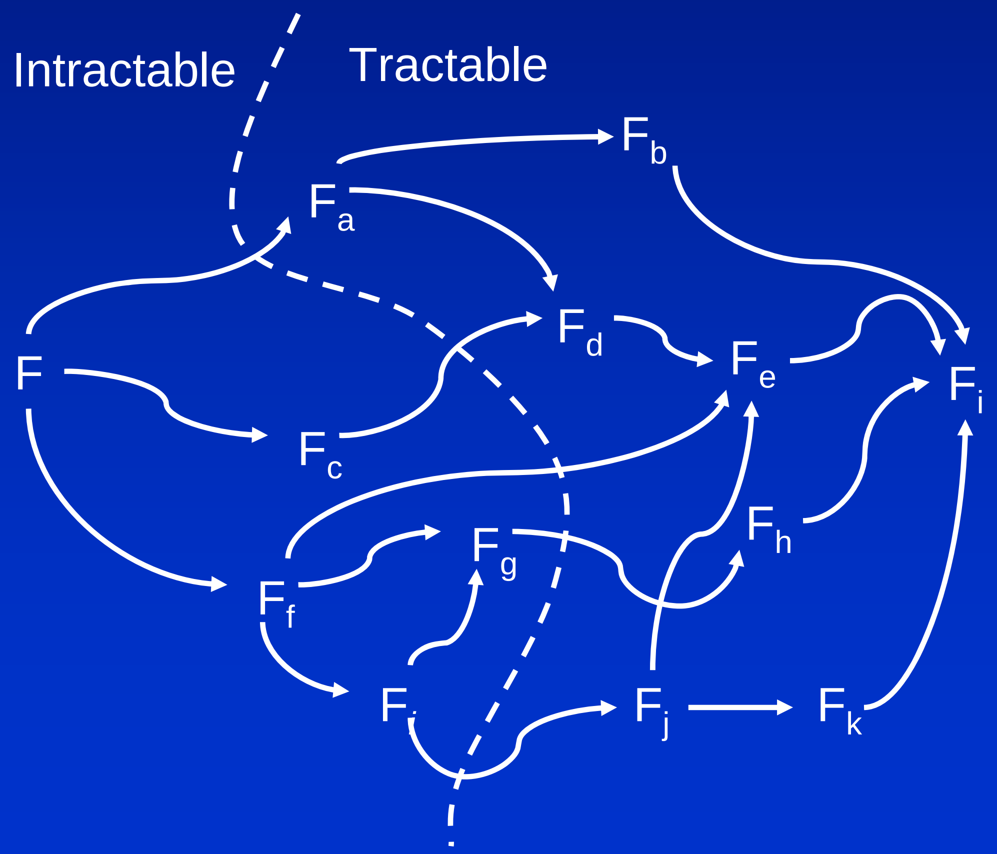
Input: Set of propositions P, set of constraints $C = C^{-} \cup C^{+}$, **such that property *X* holds.**

Output: A truth assignment to the propositions in P that satisfies a **maximum** number of constraints.

Cautions and Clarifications

Get the most out of tractability analysis!

<u>For example:</u> Analyse many (embedded) formalizations



Summary & Conclusion

Benefits of Tractable-Design Cycle

- Encourages formalization
- Helps constrain computational-level theory
- Understanding of cognitive (im)possibilities

Cautions and Clarifications

Open methodological question

- How to asses (in)tractability of theories?