Adaptive pyramid model for the Traveling Salesman Problem

Zygmunt Pizlo, Emil Stefanov & John Saalweachter
Purdue University

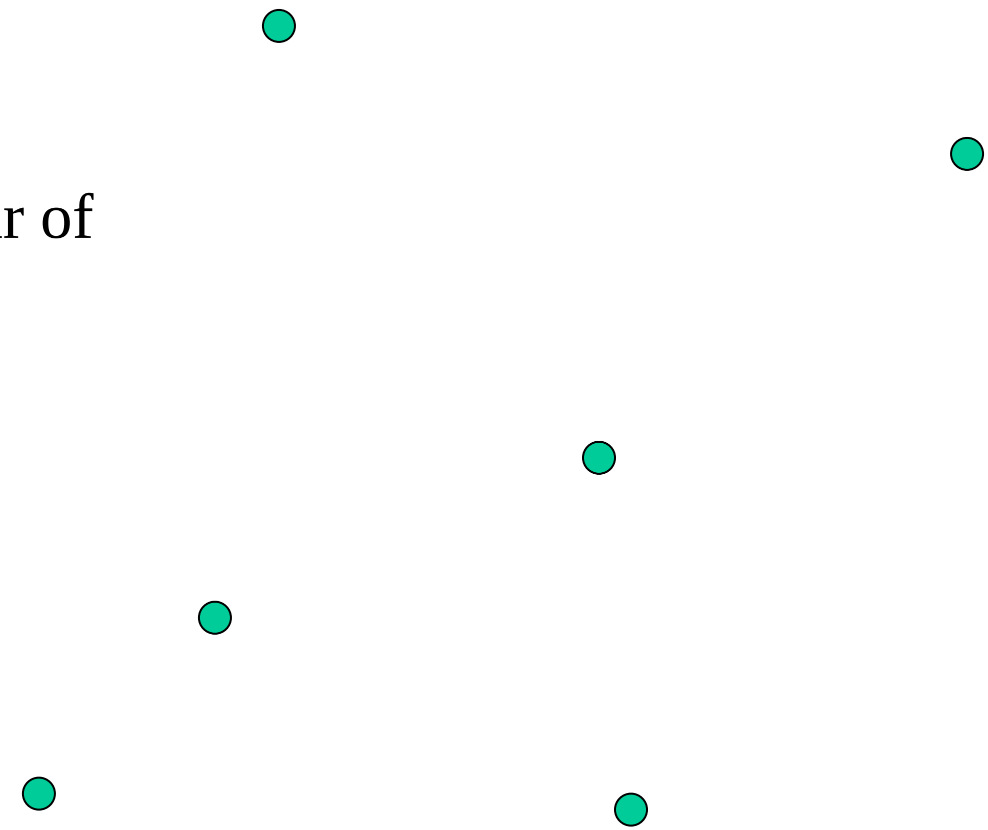
Yll Haxhimusa & Walter G. Kropatsch
Vienna University of Technology

Acknowledgment: Zheng Li

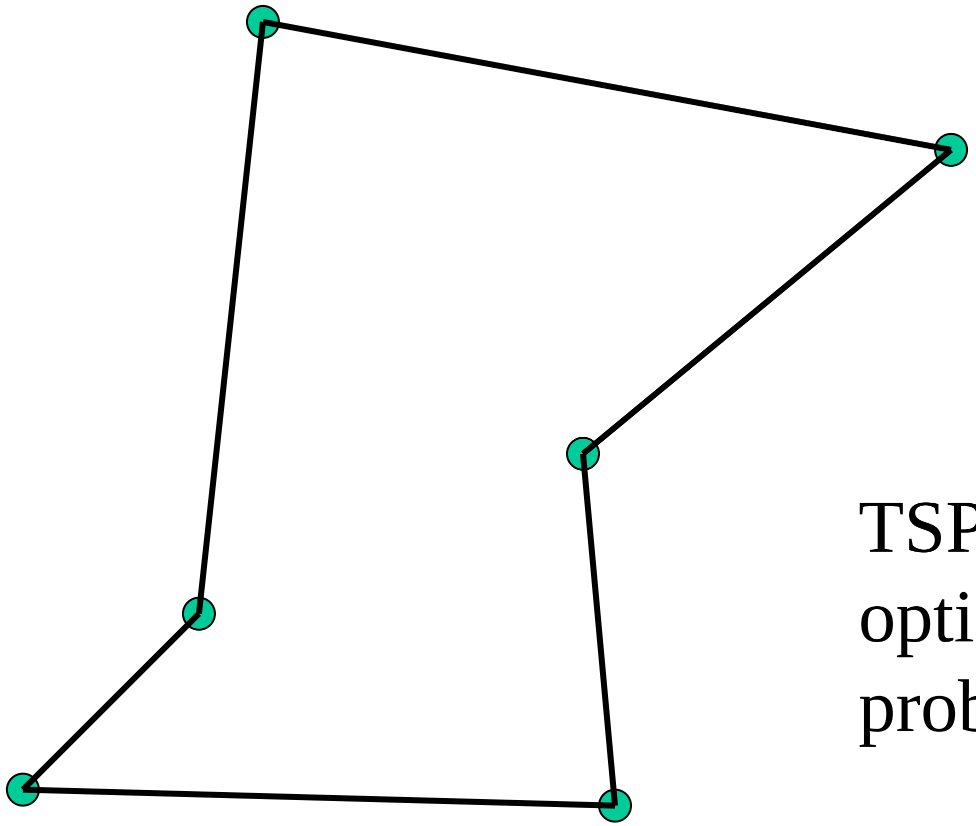
Support: AFOSR

Traveling Salesman Problem:

Find the shortest tour of N cities.



Traveling Salesman Problem:

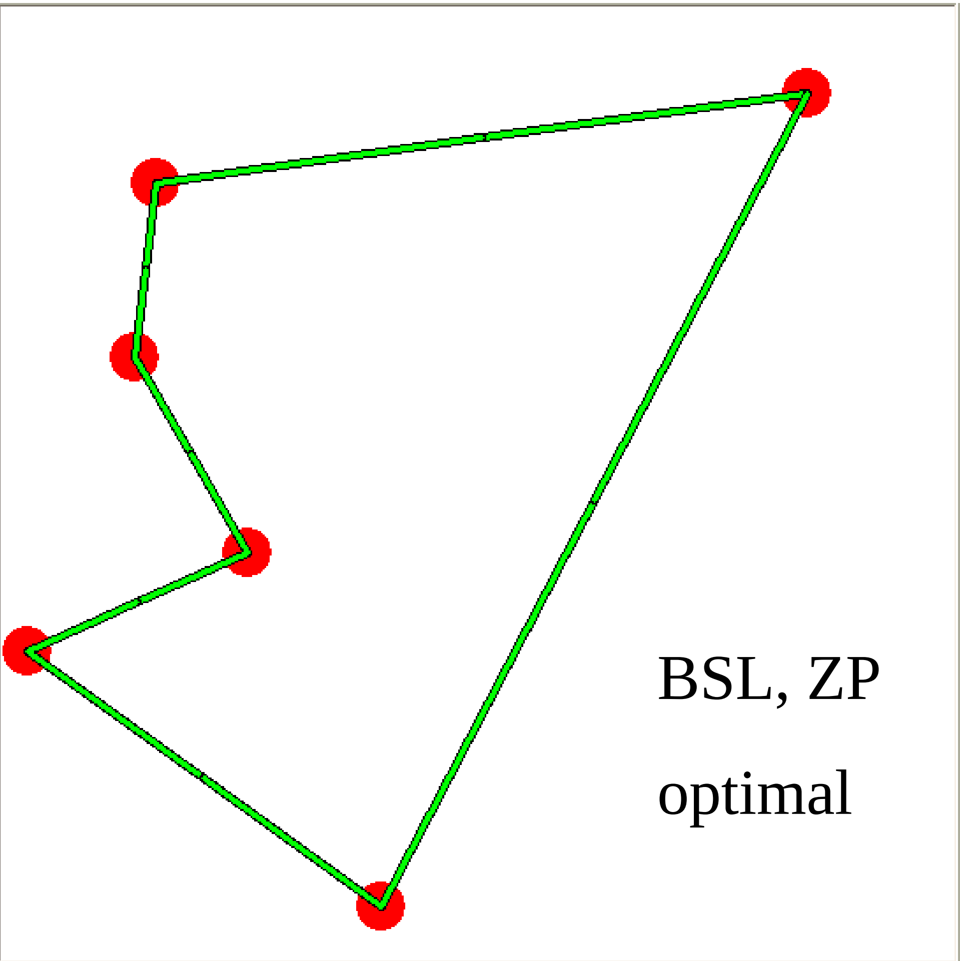


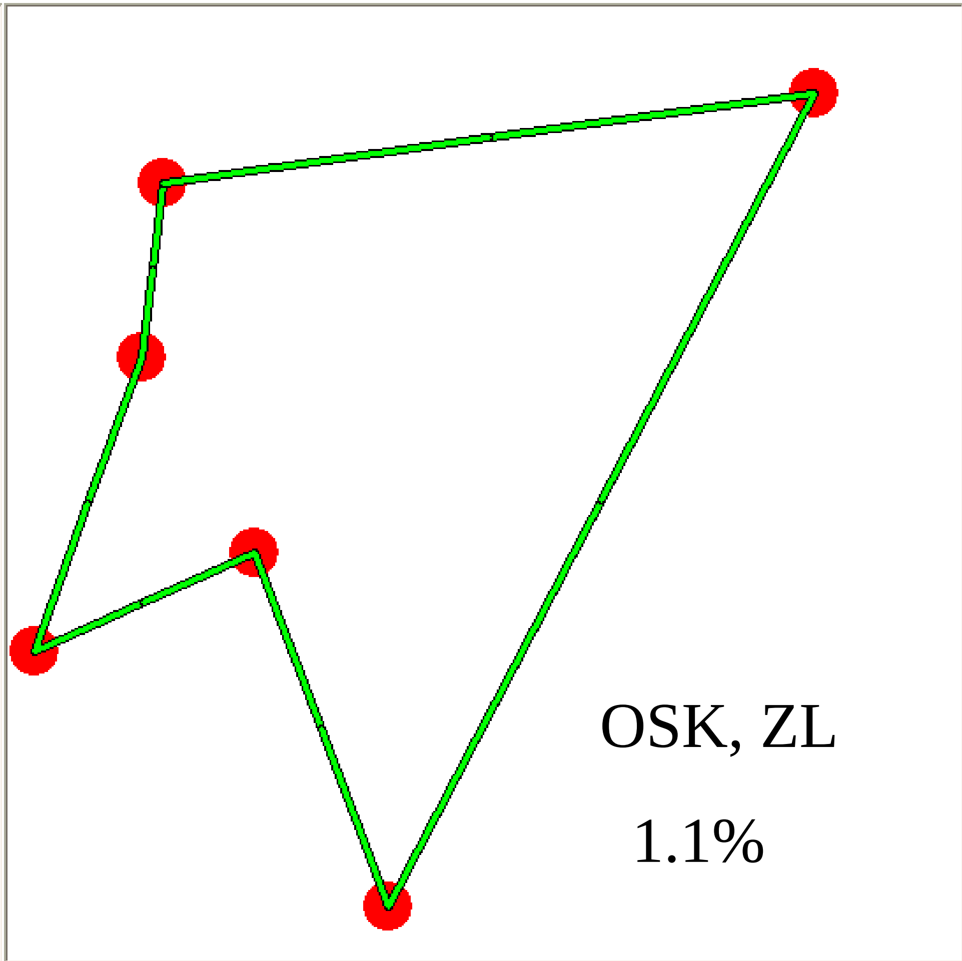
TSP is a difficult optimization problem.

Experiment

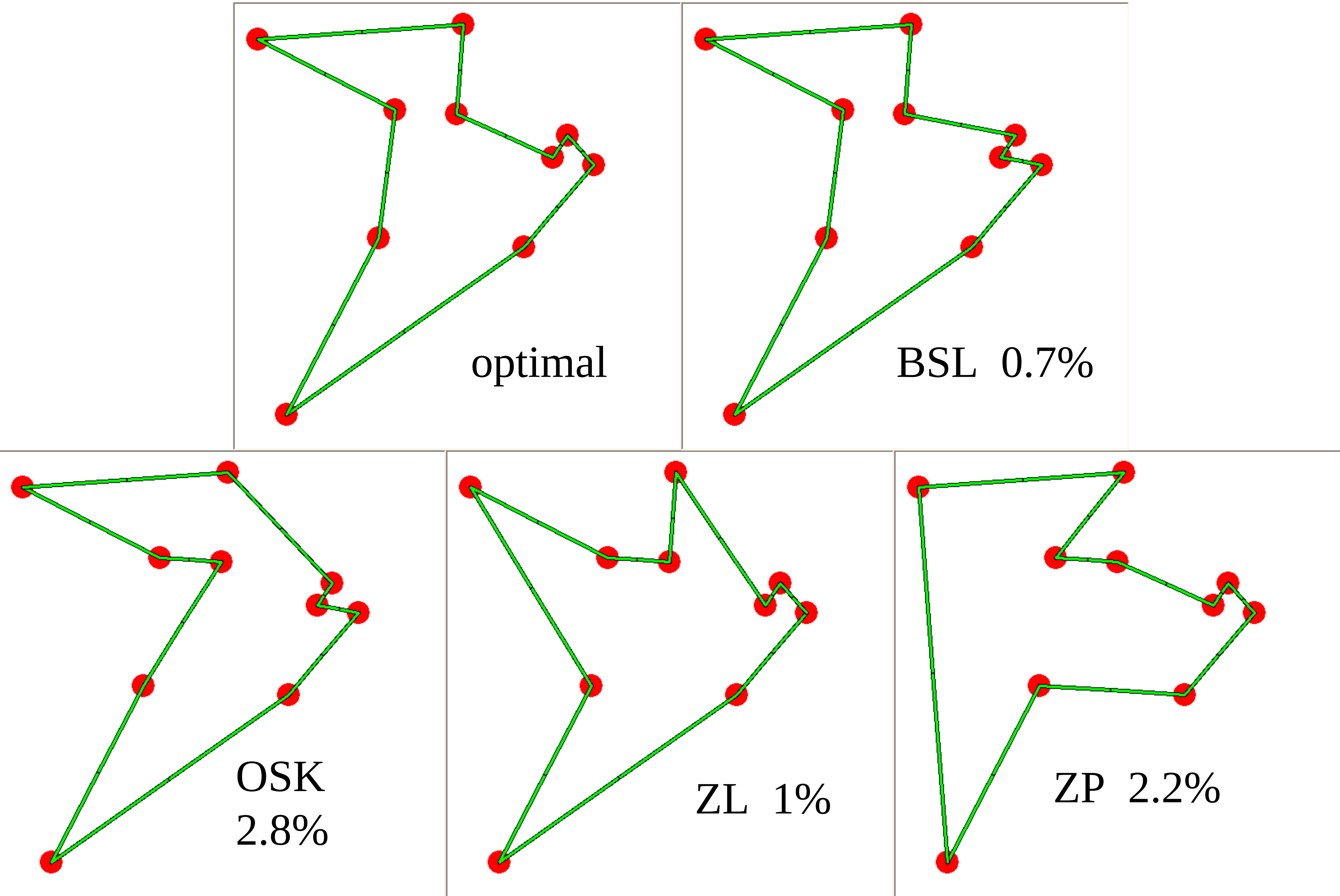
- 5 subjects
- Problem size: 6, 10, 20, 50
- 25 random problems per size
- Problems were shown on a computer screen

6 city

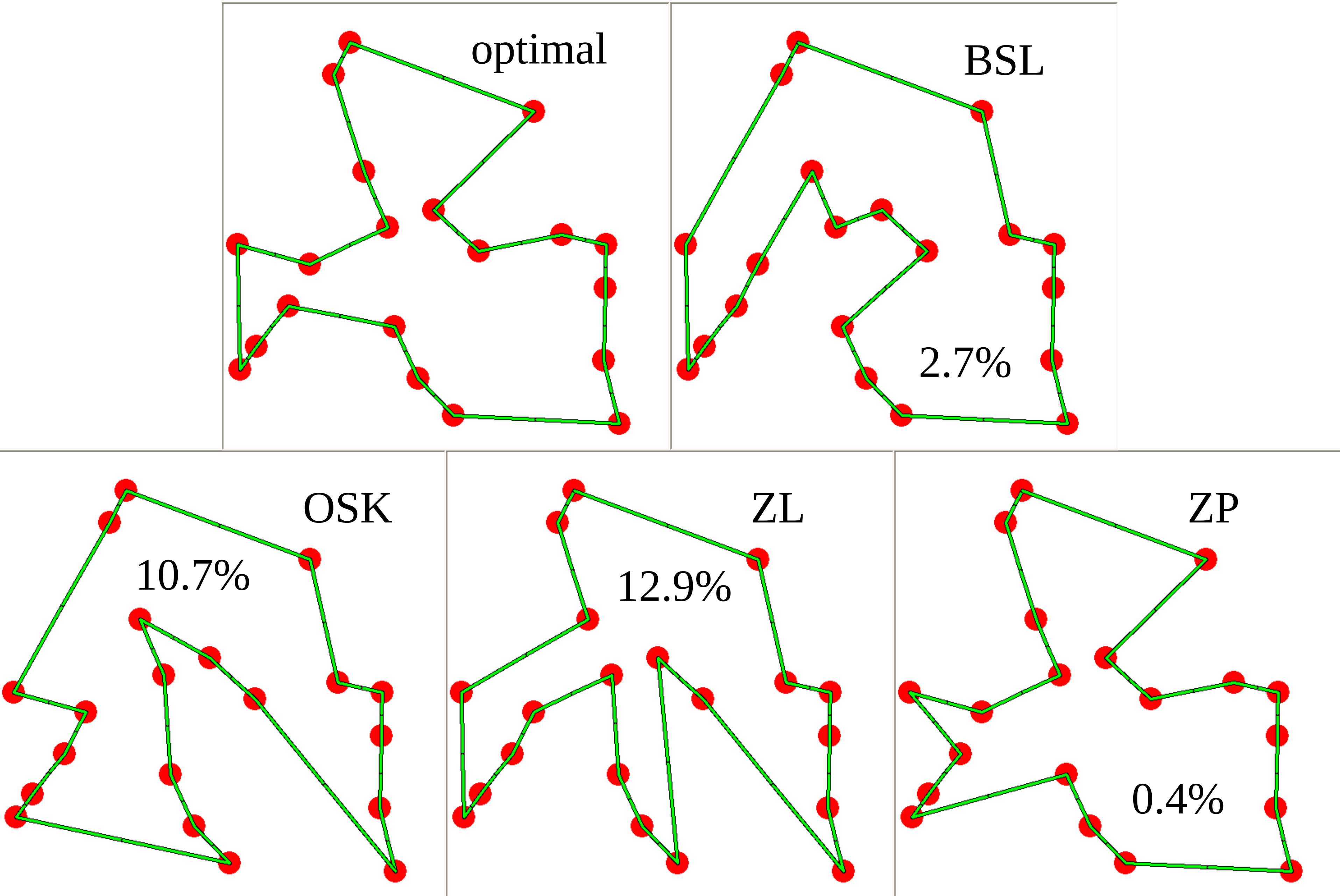




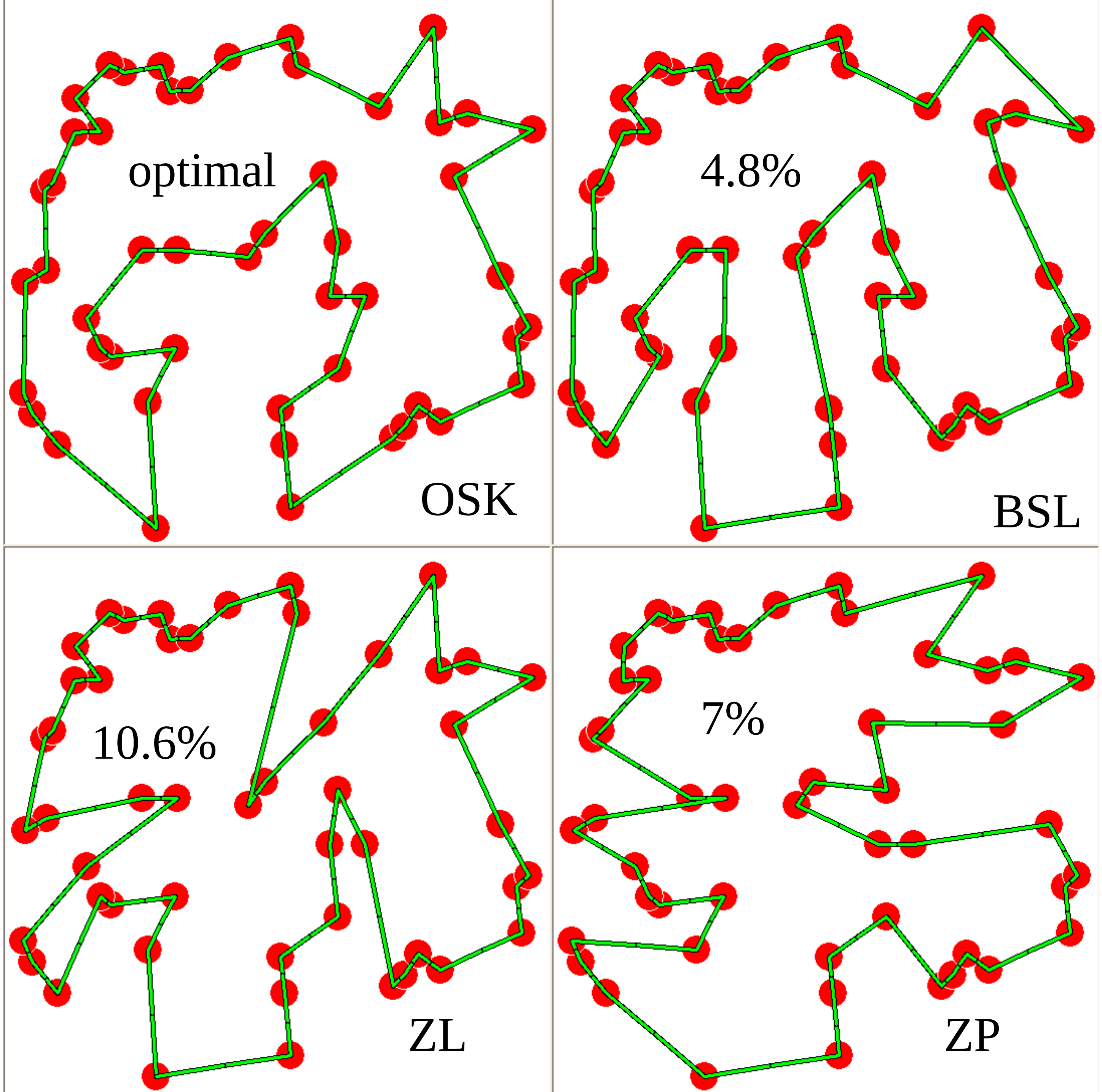
10 city

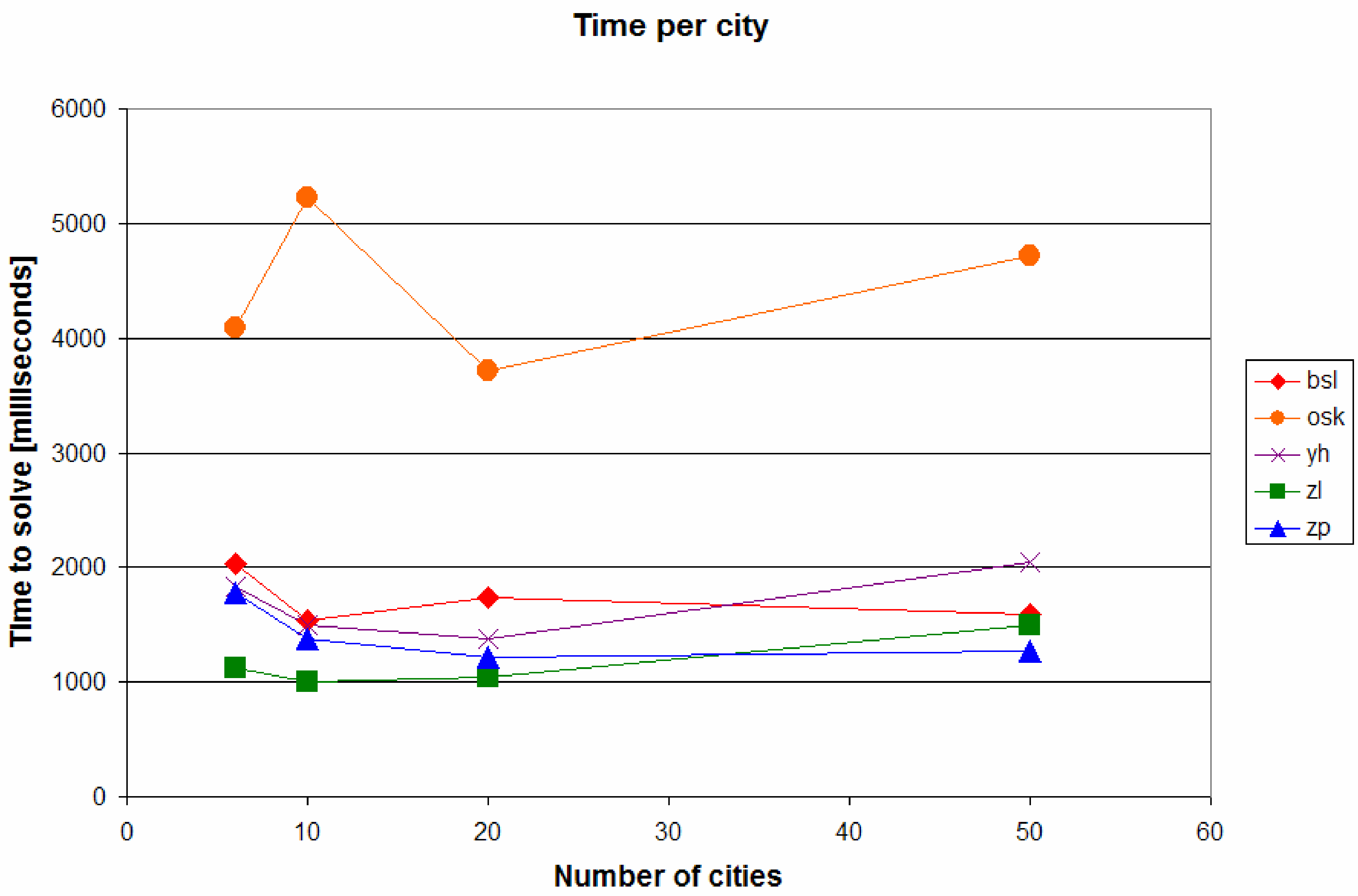


20 city

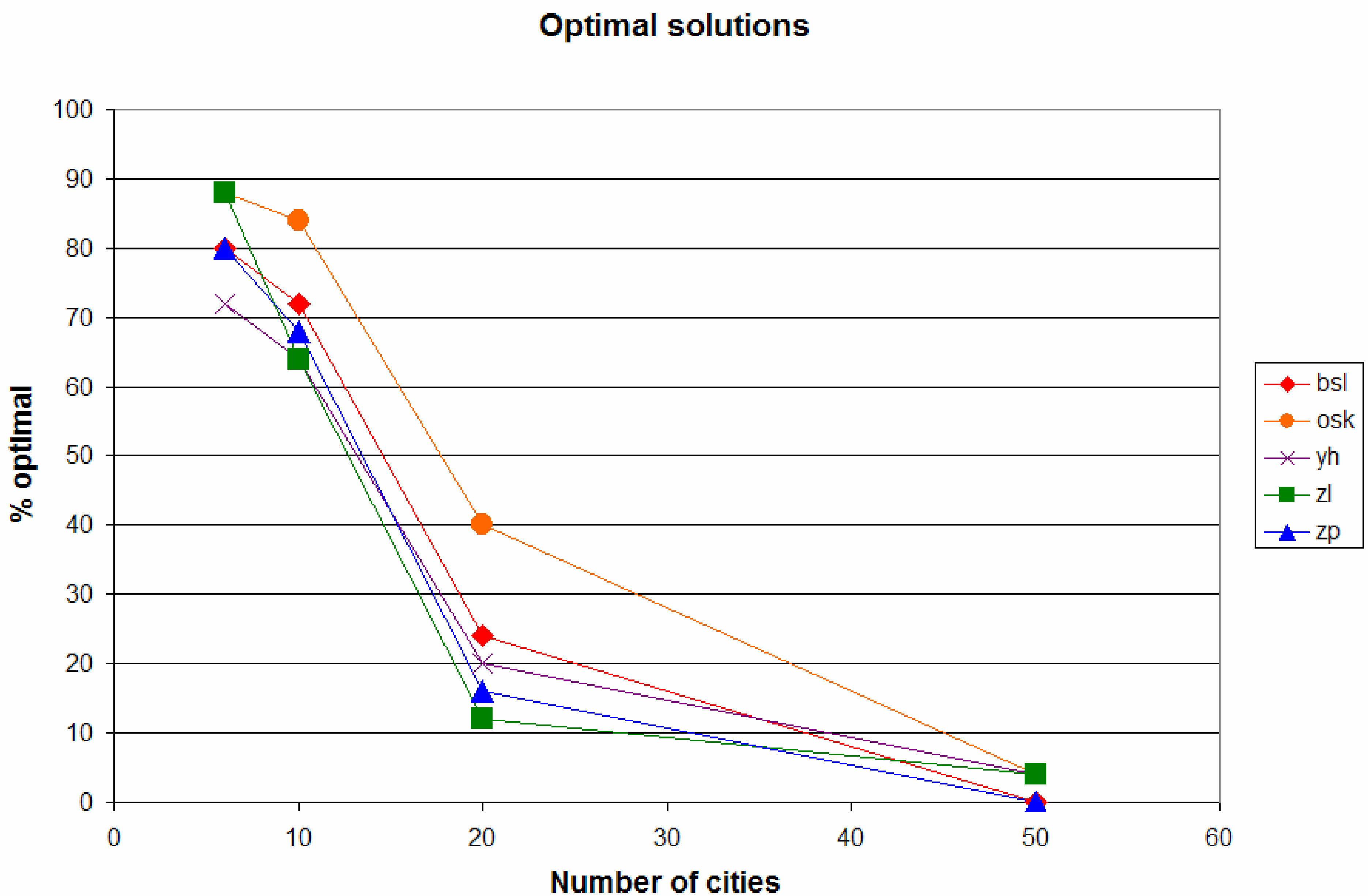


50
city

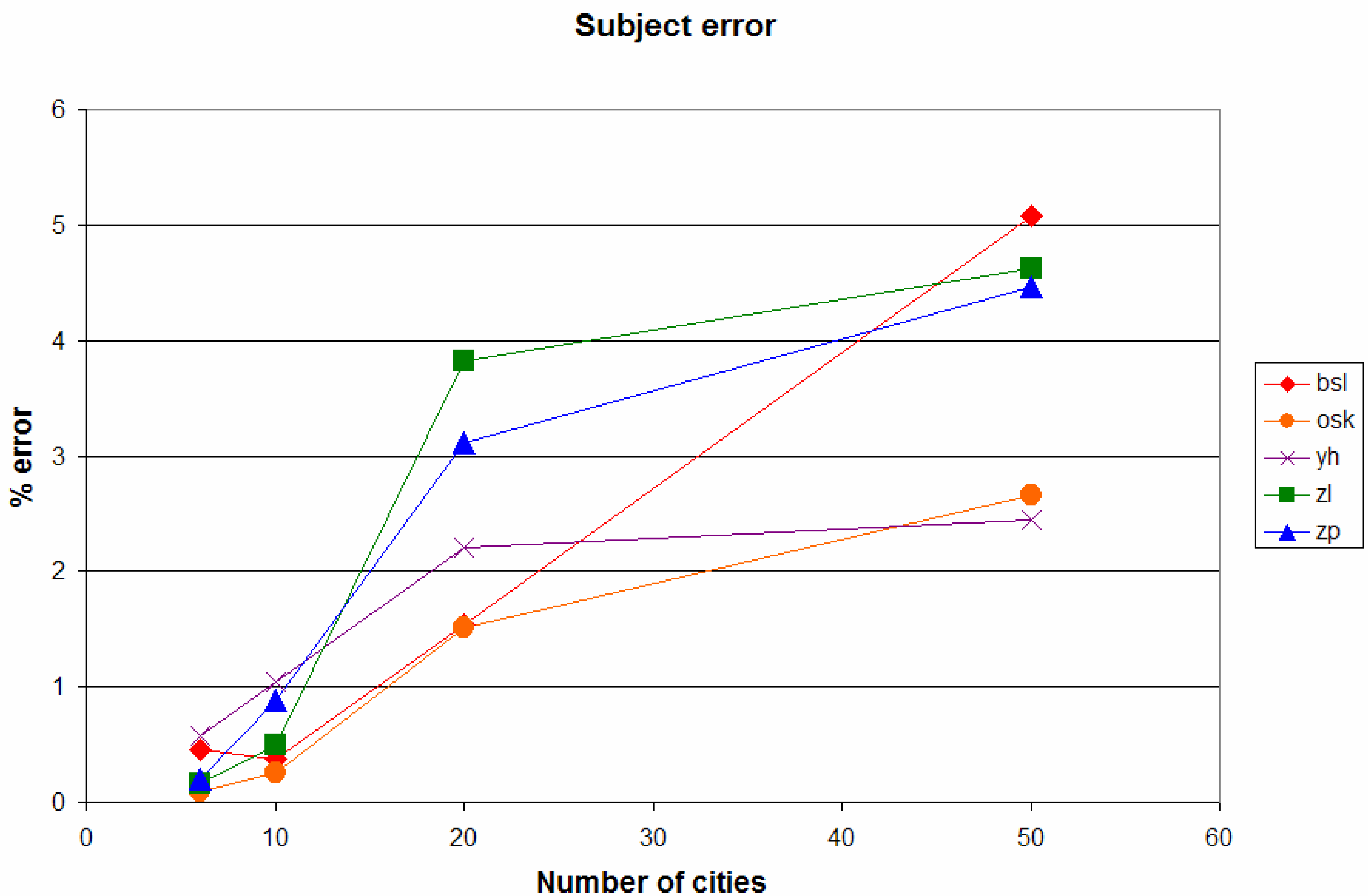


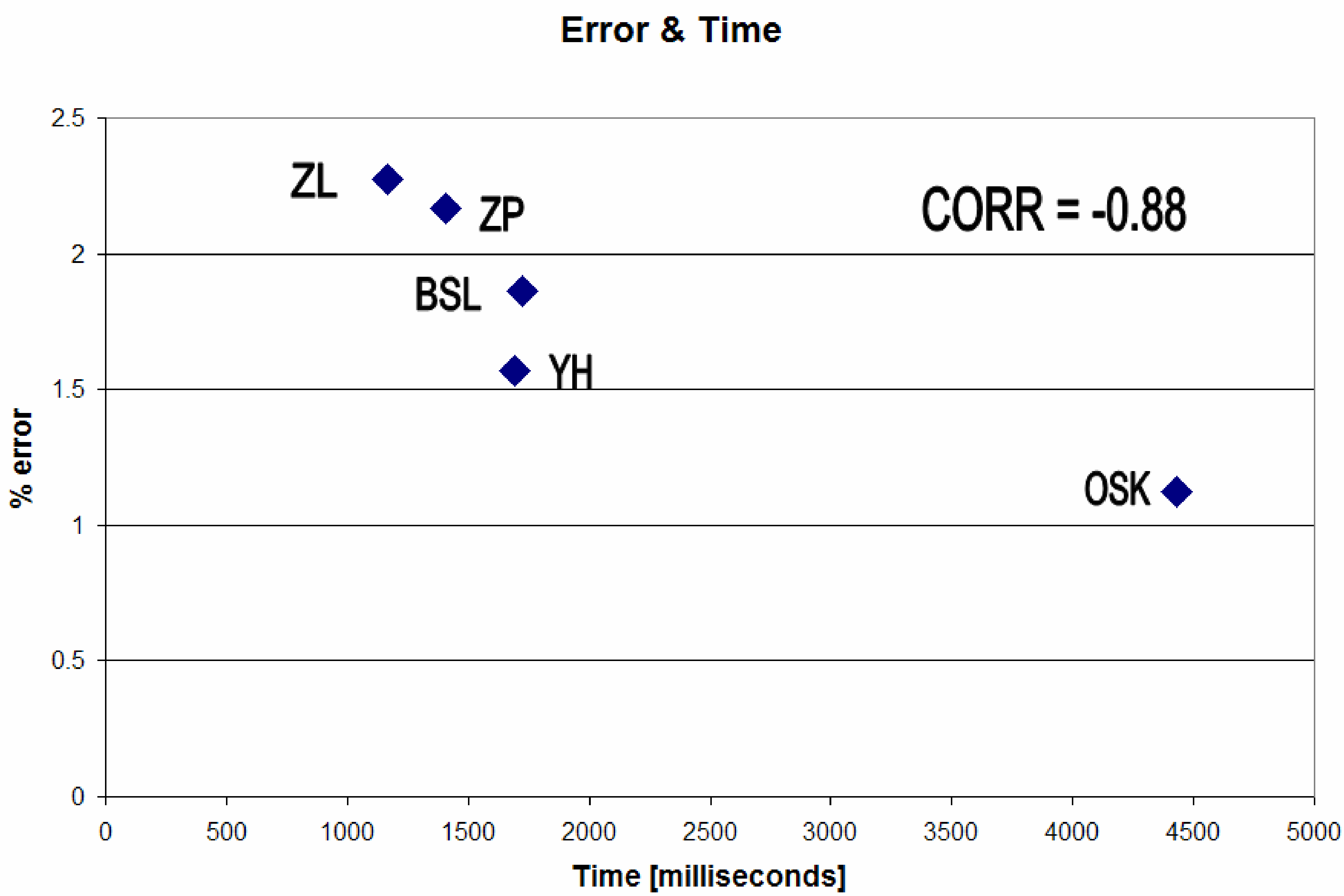


Time per city
Time to solve [milliseconds]
6000
5000
4000
3000
2000
1000
0
0
10
20
30
40
50
60
Number of cities
bsl
osk
yh
zl
zp



Optimal solutions
% optimal
100
90
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
Number of cities
bsl
osk
yh
zl
zp


Subject error
% error
Number of cities
0
1
2
3
4
5
6
0
10
20
30
40
50
60
bsl
osk
yh
zl
zp

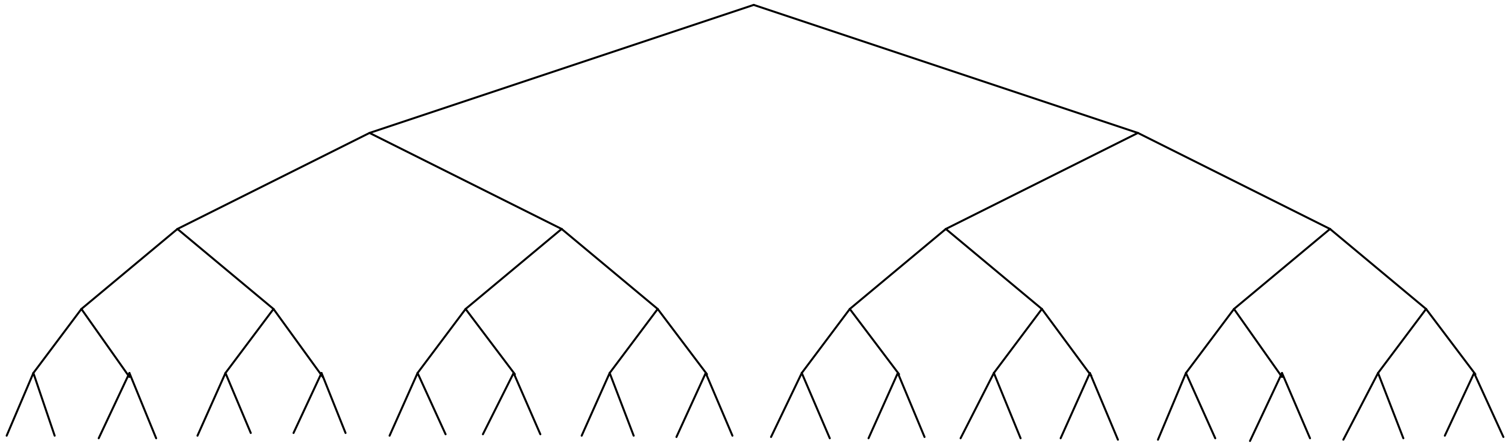


Error & Time
% error
2.5
2
1.5
1
0.5
0
ZL
ZP
BSL
YH
OSK
CORR = -0.88
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
Time [milliseconds]

Model

- Multiresolution pyramid representation
- Top-down process of tour approximations

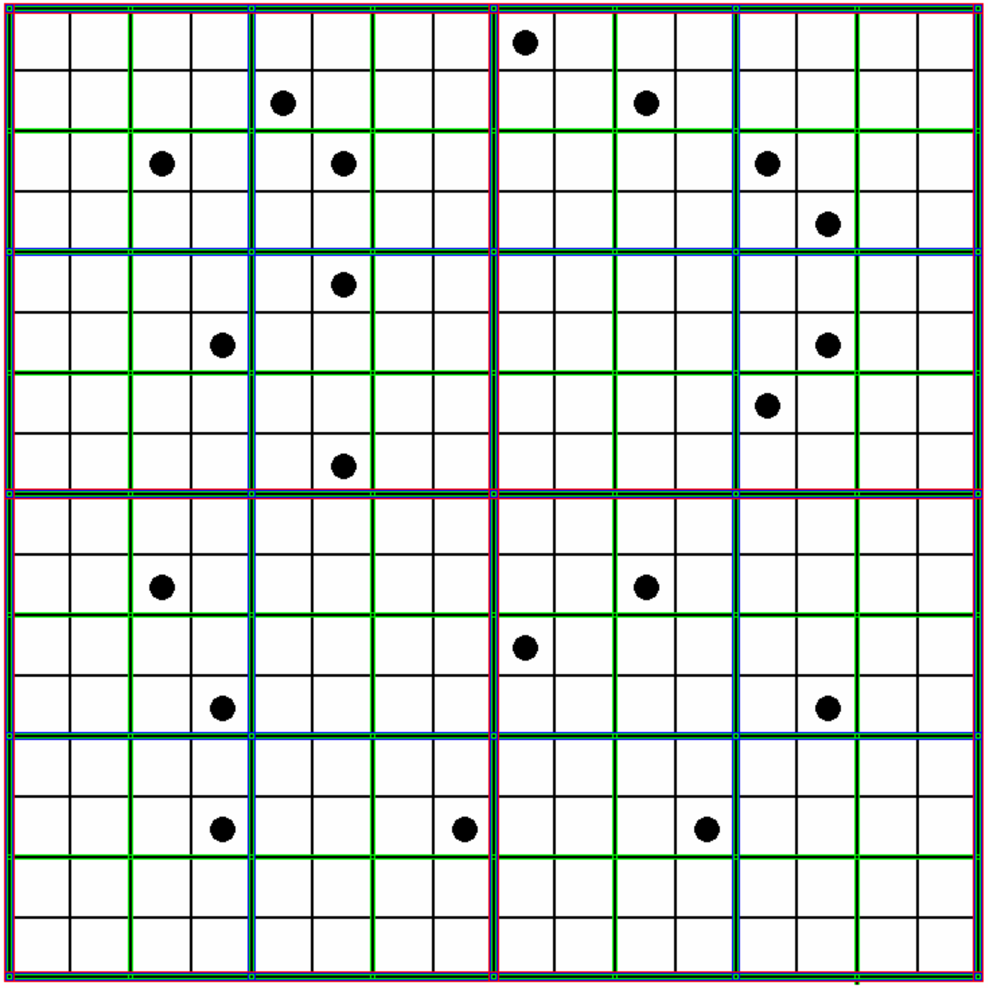
1D Pyramid architecture



The number of nodes on layer i+1 is b times smaller than that on layer i. Receptive field on layer i+1 is b times larger than that on layer i.

What is local close to the top, is global close to the bottom.

2D Pyramid Representation

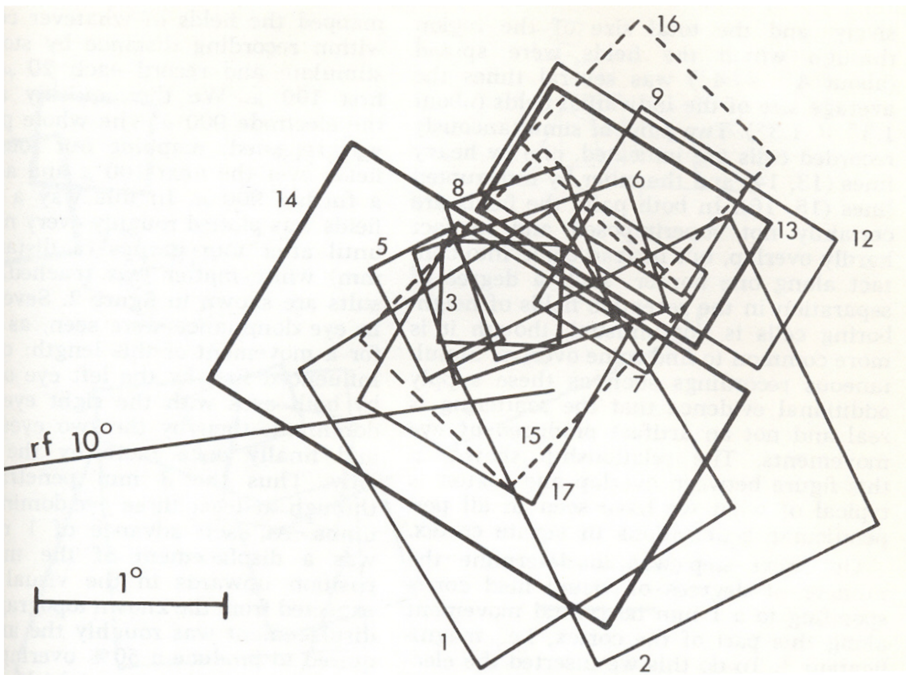


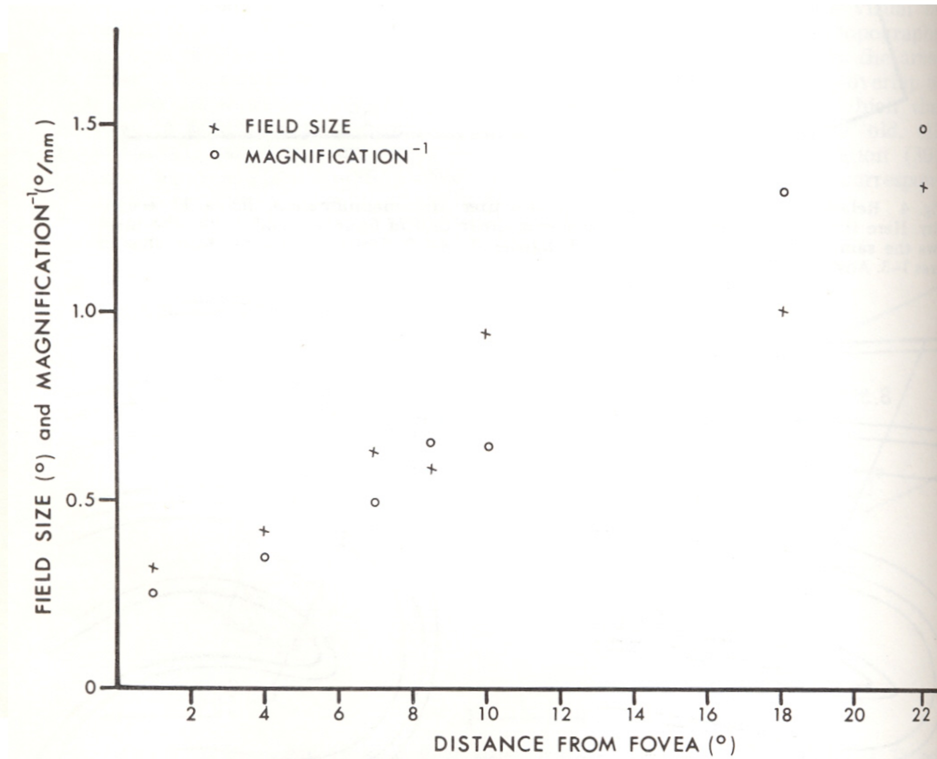
Model

- Multiresolution pyramid representation
- Top-down process of tour approximations
- Pyramid with the “fovea” and with “eye movements”

Neuroanatomy of the visual system

(Hubel & Wiesel, 1974)

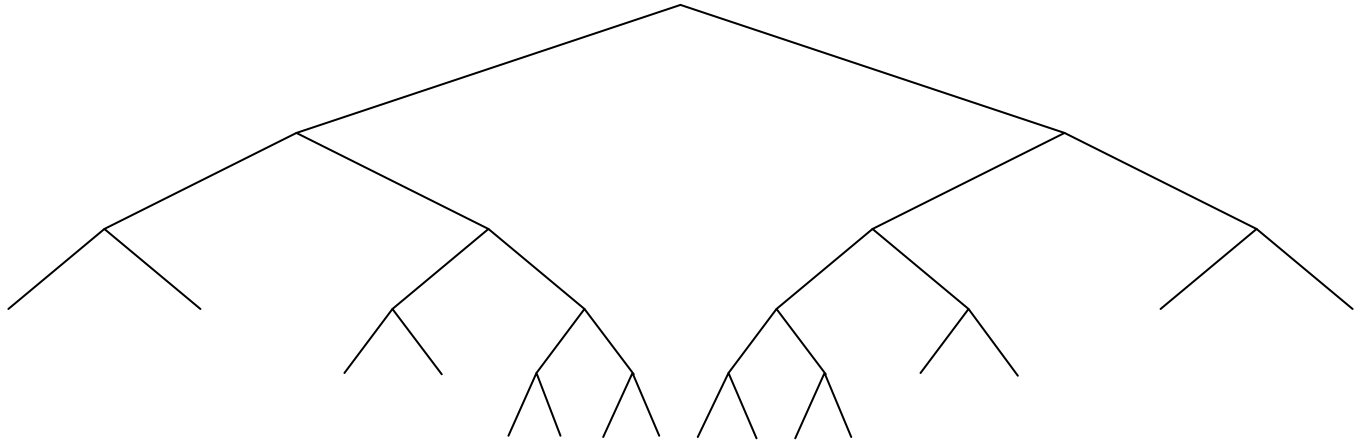




At each retinal location, there is a family of receptive fields with different sizes and resolutions.

The size of the smallest field is a function of eccentricity.

Pyramid with Fovea



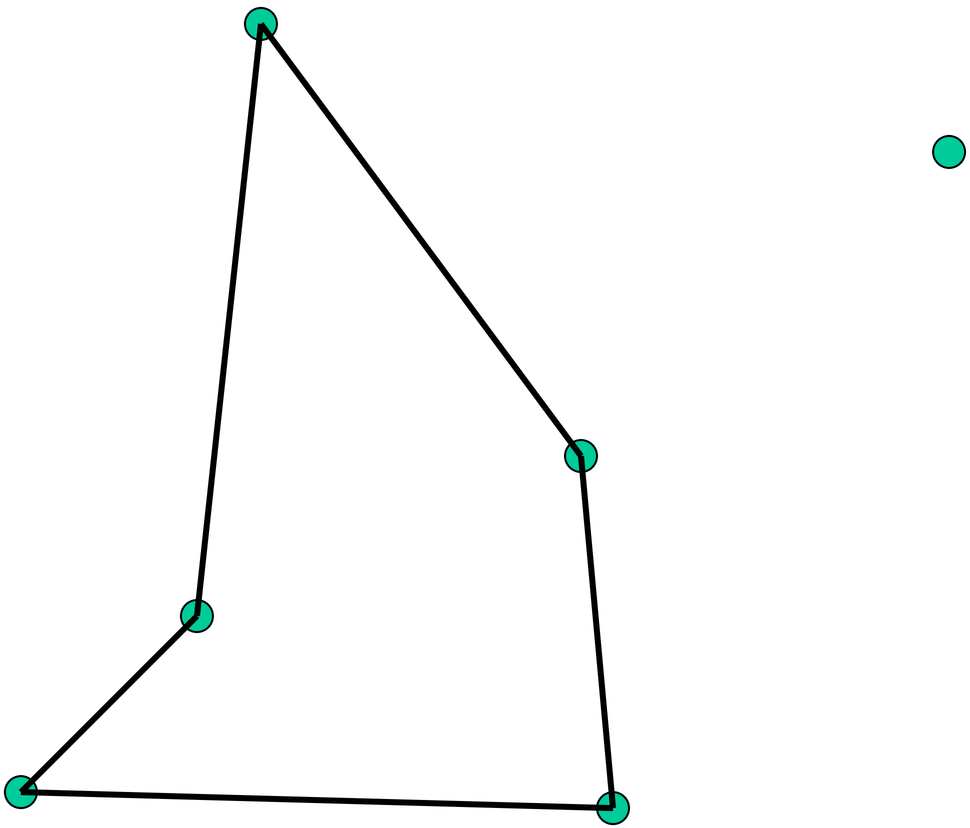
Resolution of the finest representation decreases with the distance from the fixation point – this corresponds to the non-uniform density of the receptors on the retina.

Prevents from handling too much information at a time.

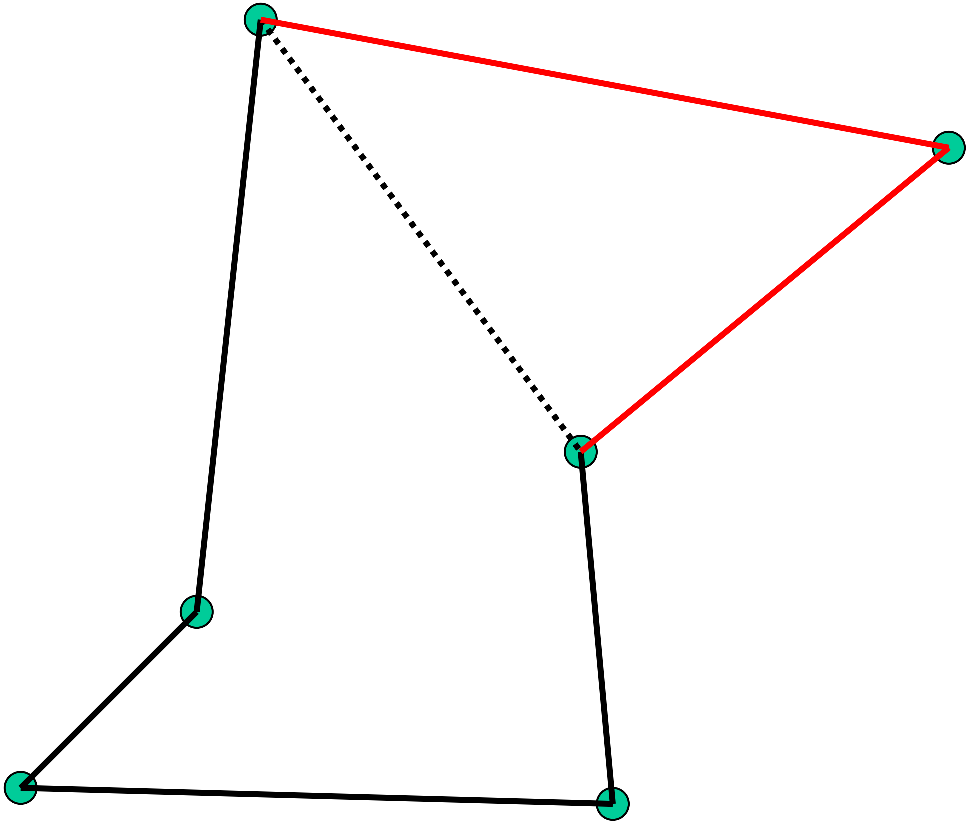
Model

- Multiresolution pyramid representation
- Top-down process of tour approximations
- Pyramid with the “fovea” and with “eye movements”
- Local search by means of “cheapest insertion”

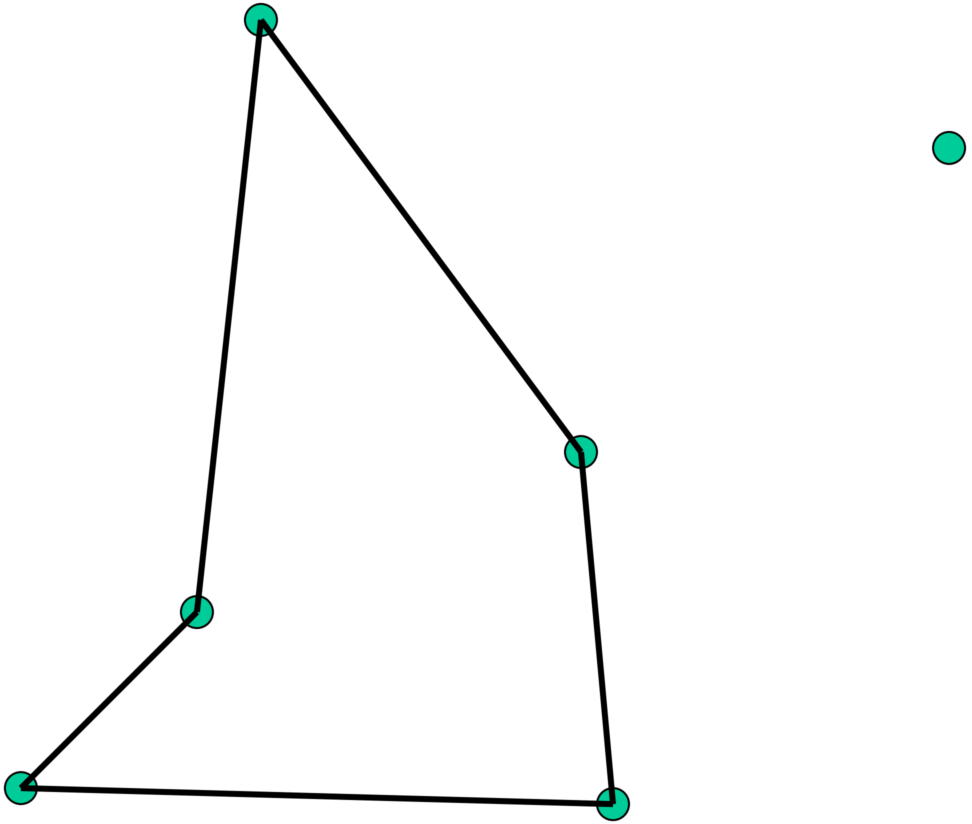
Cheapest Insertion



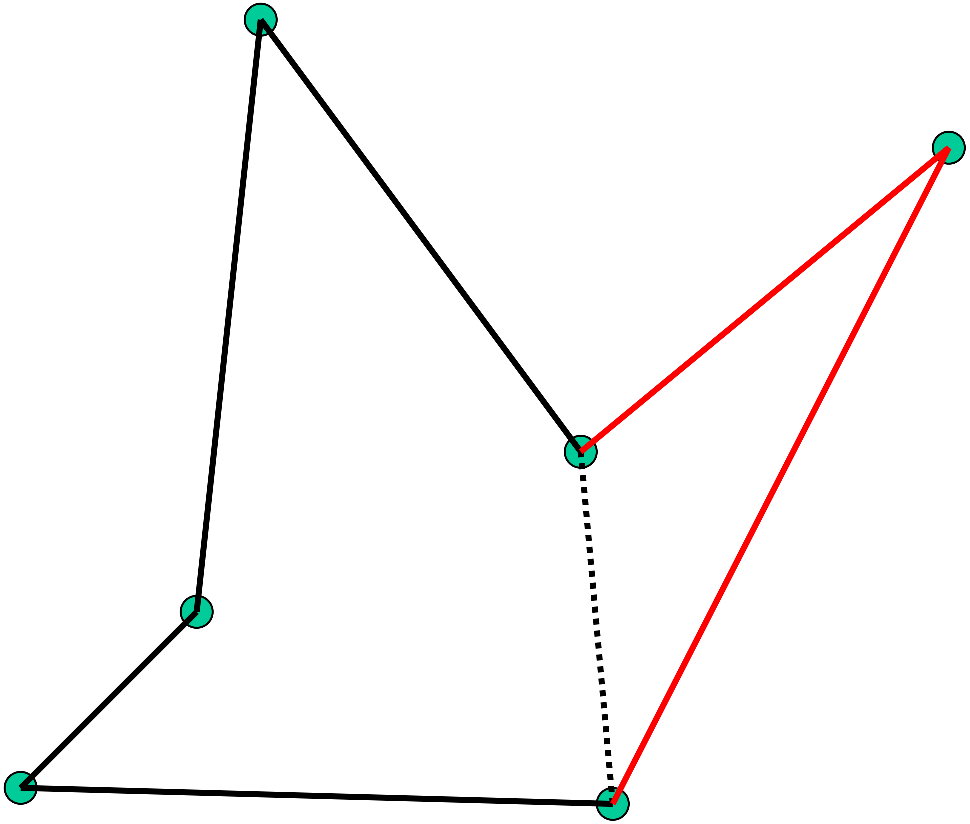
Cheapest Insertion



Cheapest Insertion



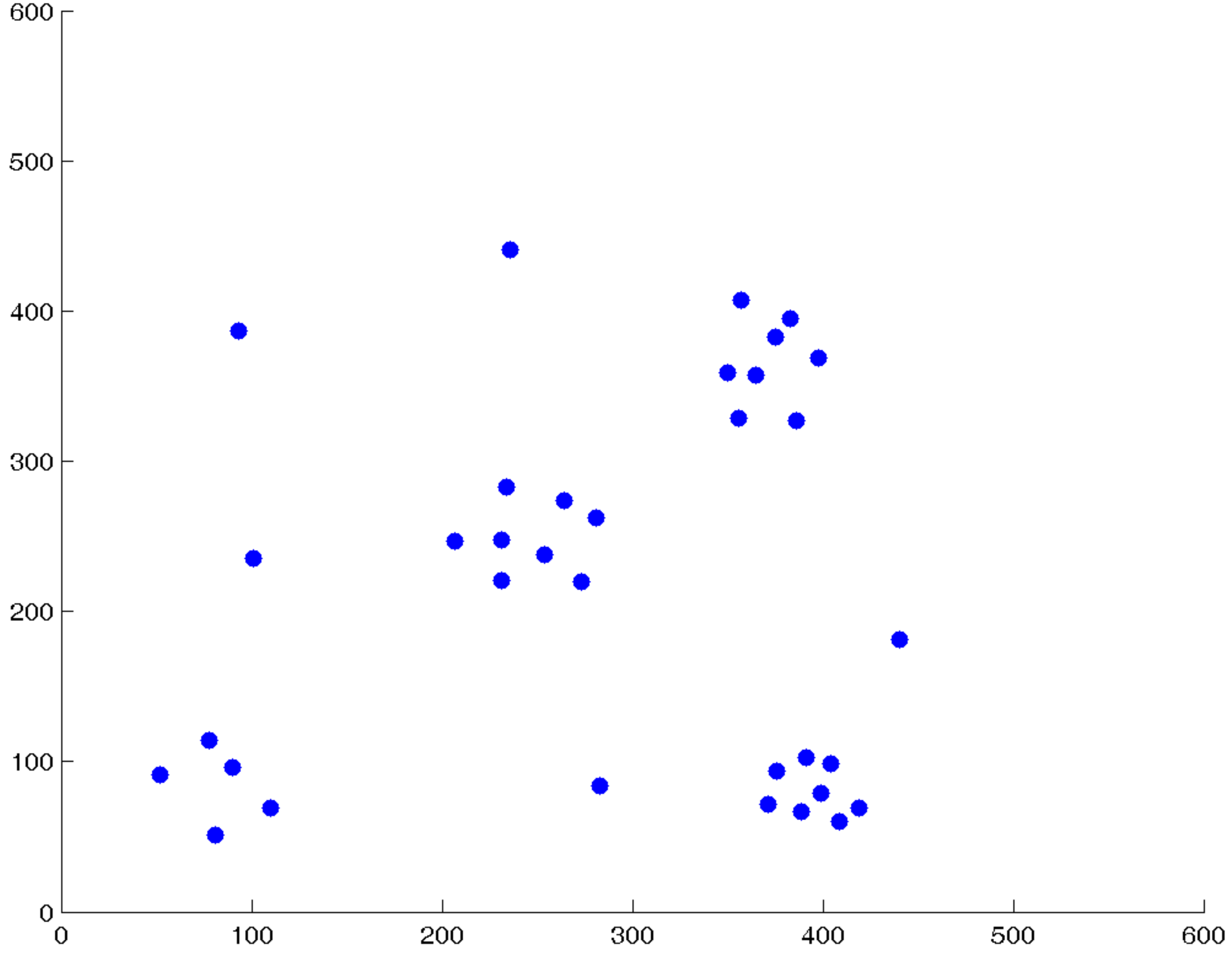
Cheapest Insertion

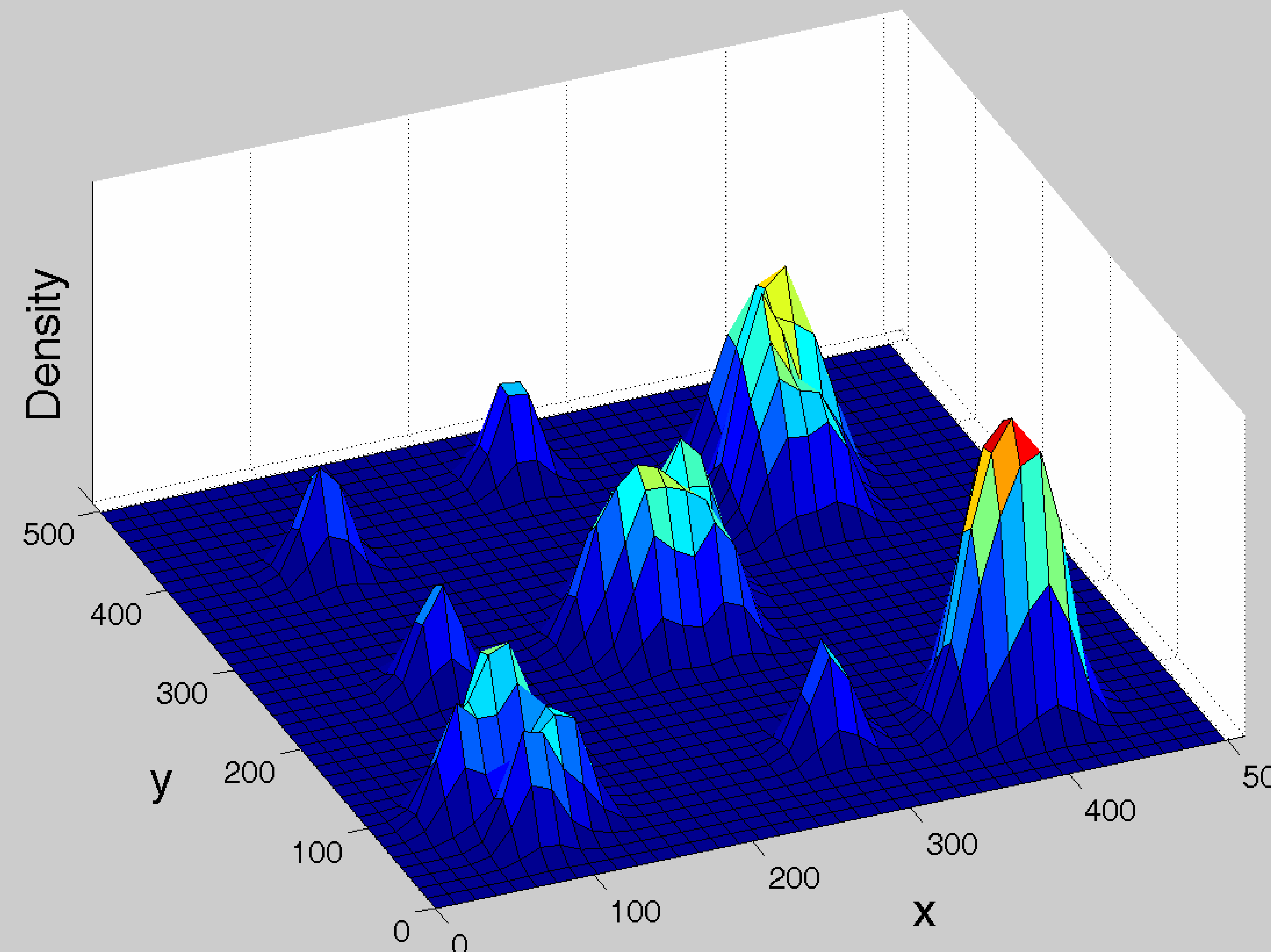


Model

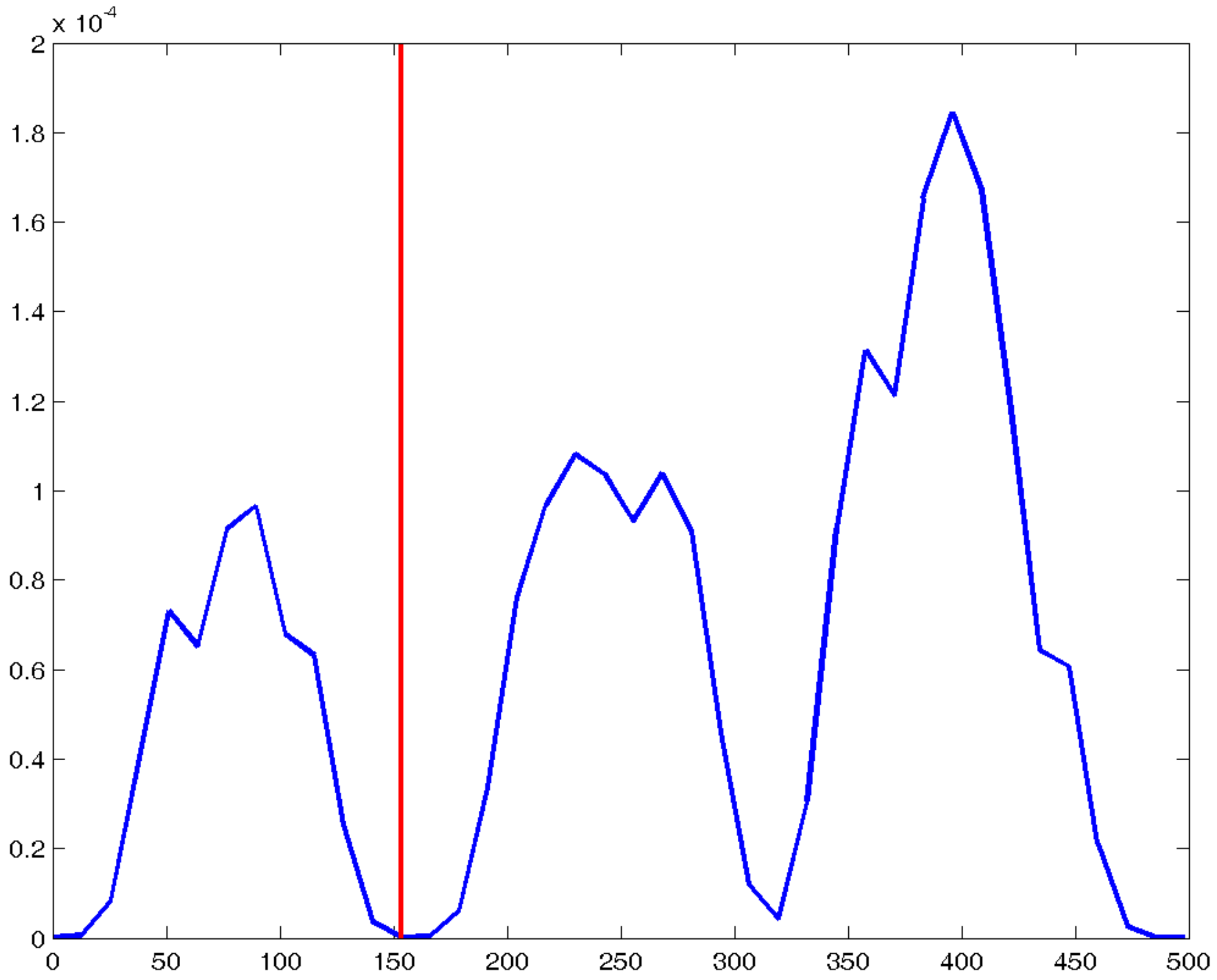
- Multiresolution pyramid representation
- Top-down process of tour approximations
- Pyramid with the “fovea” and with “eye movements”
- Local search by means of “cheapest insertion”
- Adaptive receptive fields

Blurring with Gaussian Filter

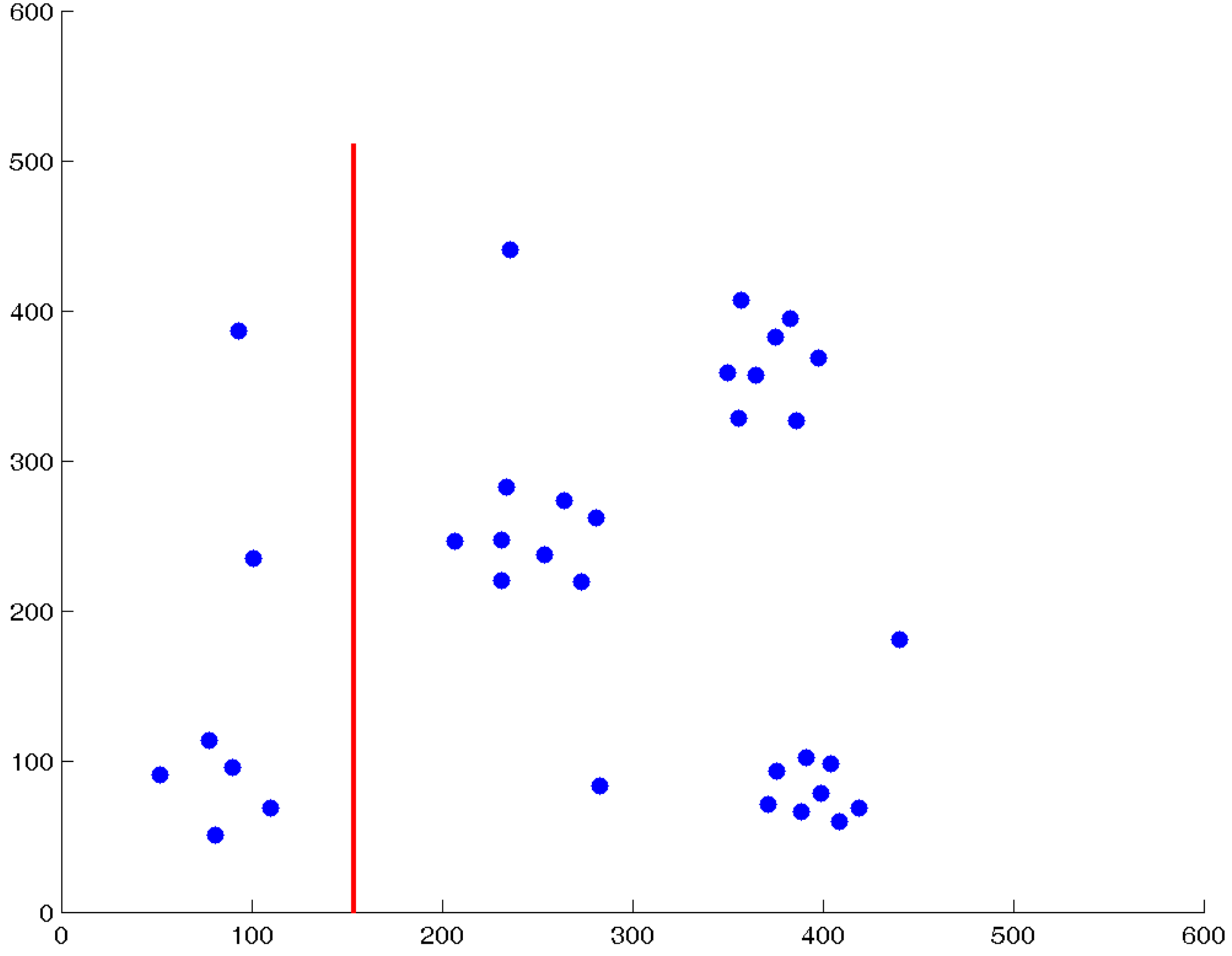



Density
y
x
0
100
200
300
400
500
0
100
200
300
400
50

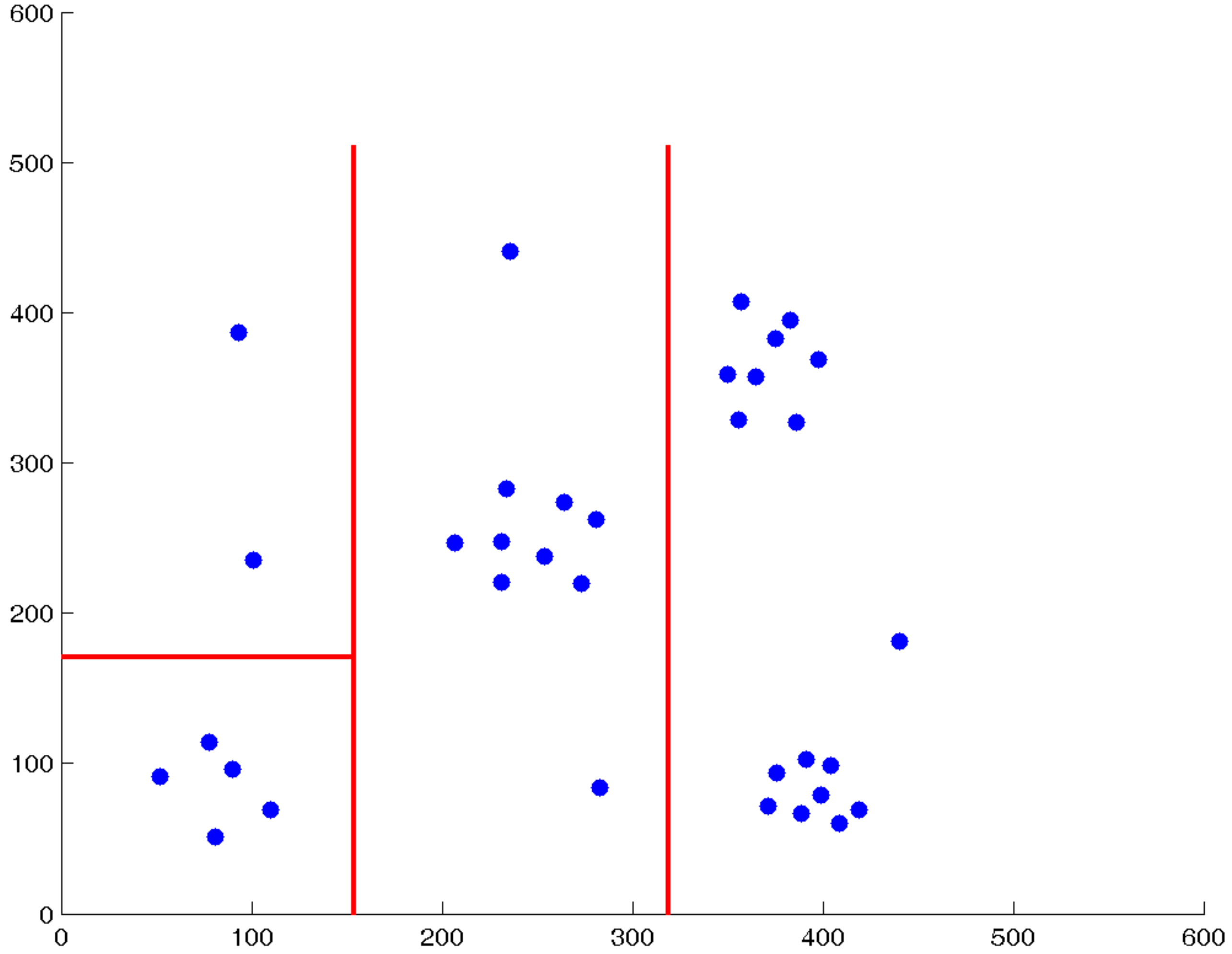
Min-Max Method for Determining Cluster Boundaries



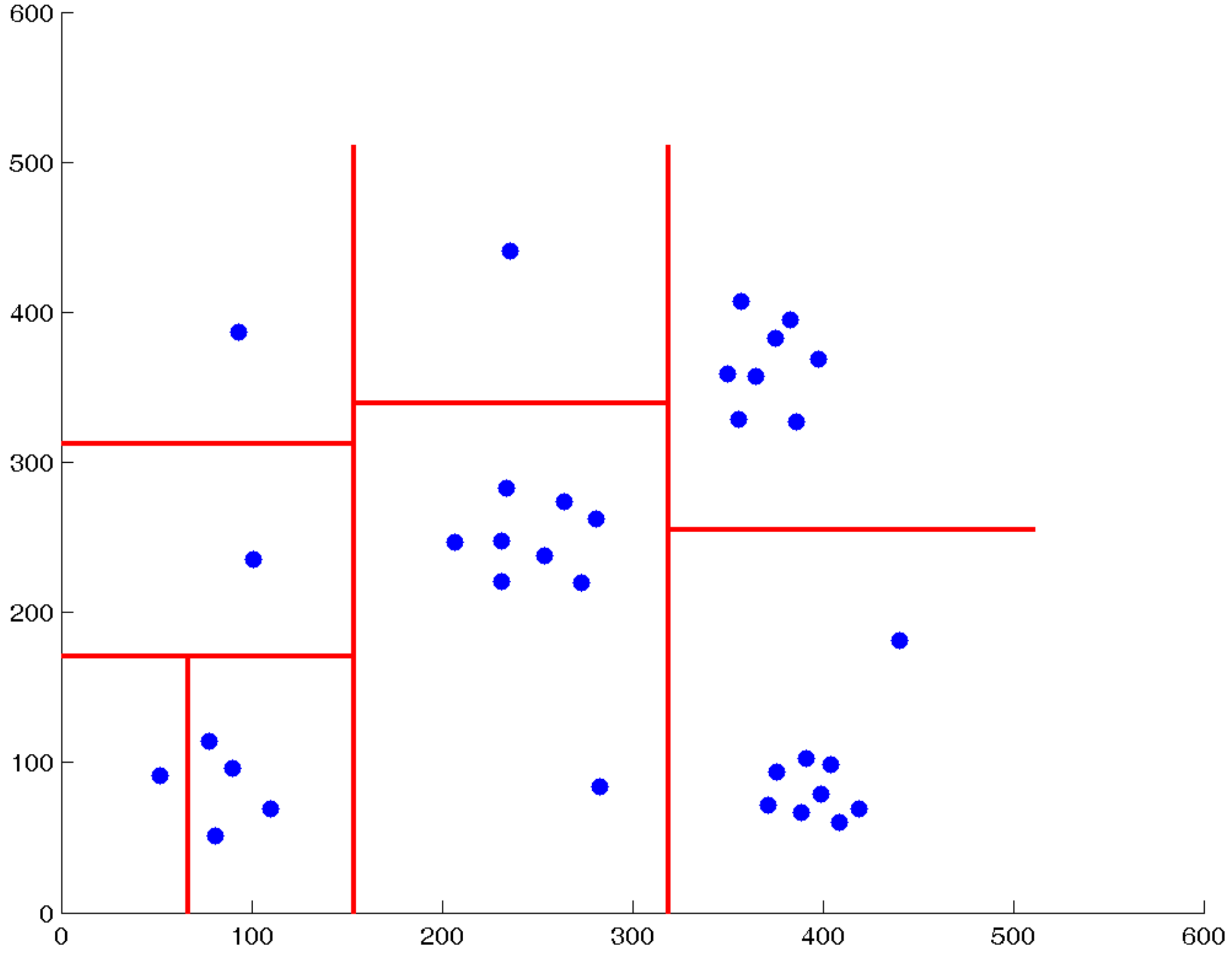
Bisection Pyramid – Top Layer (8)



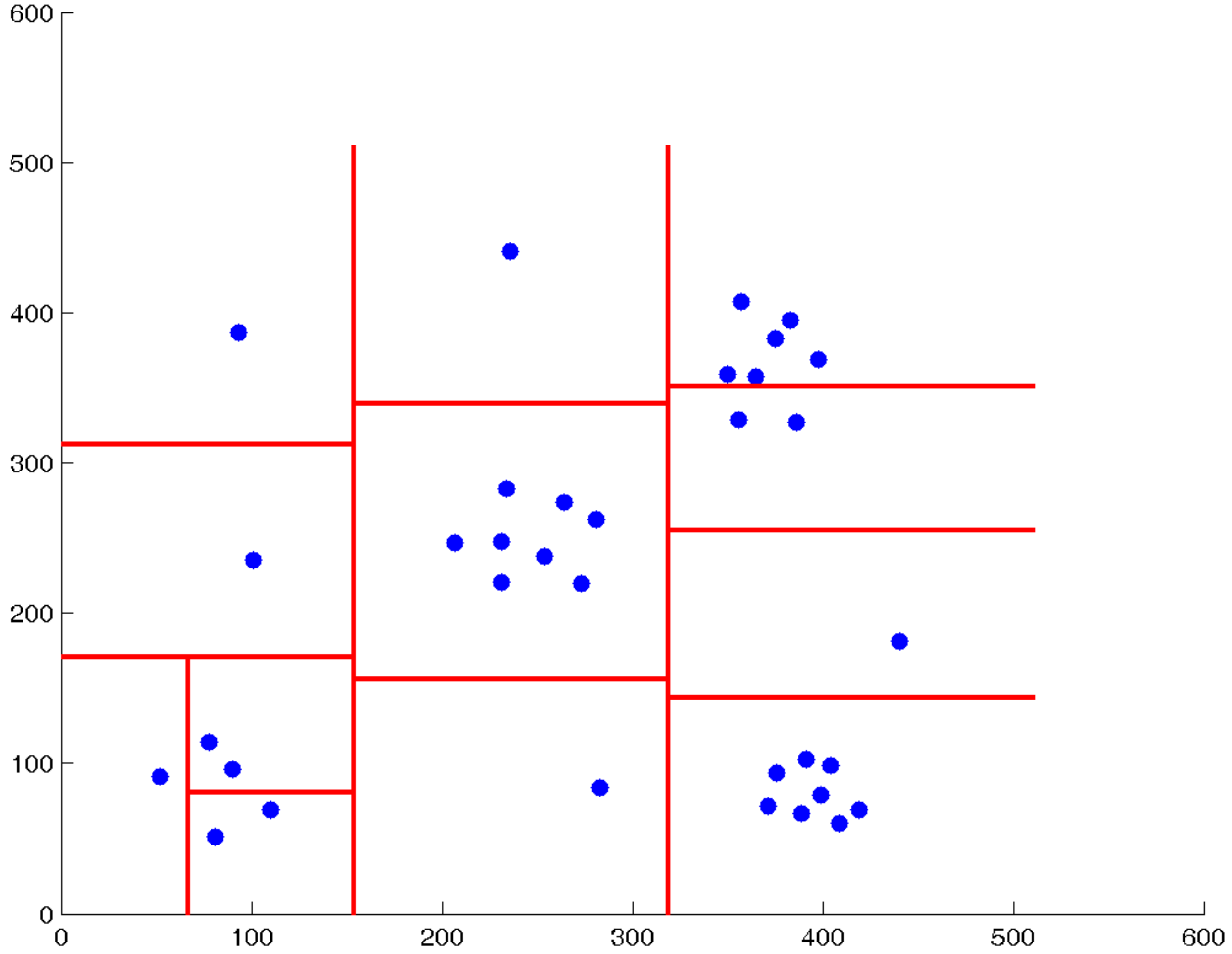
Layer 7



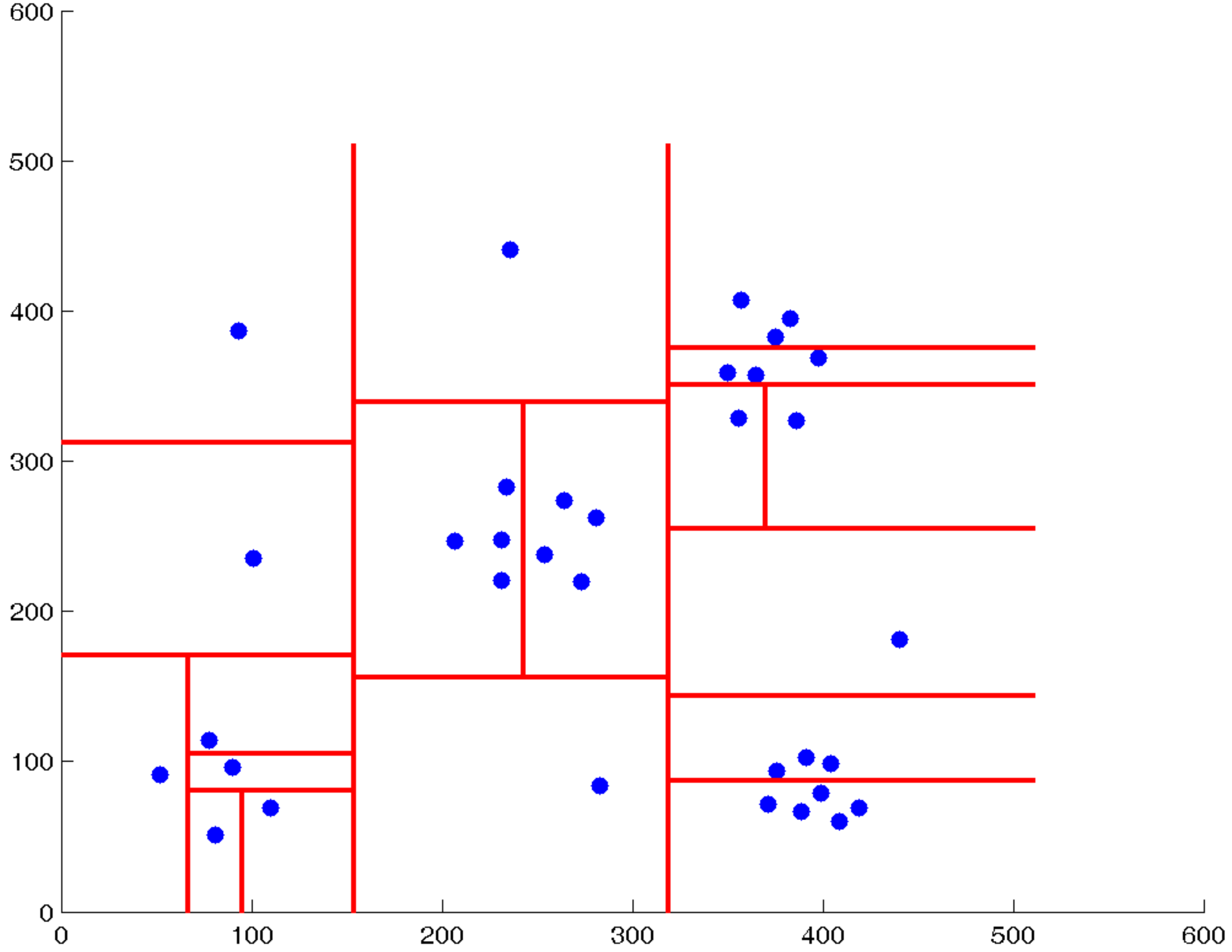
Layer 6



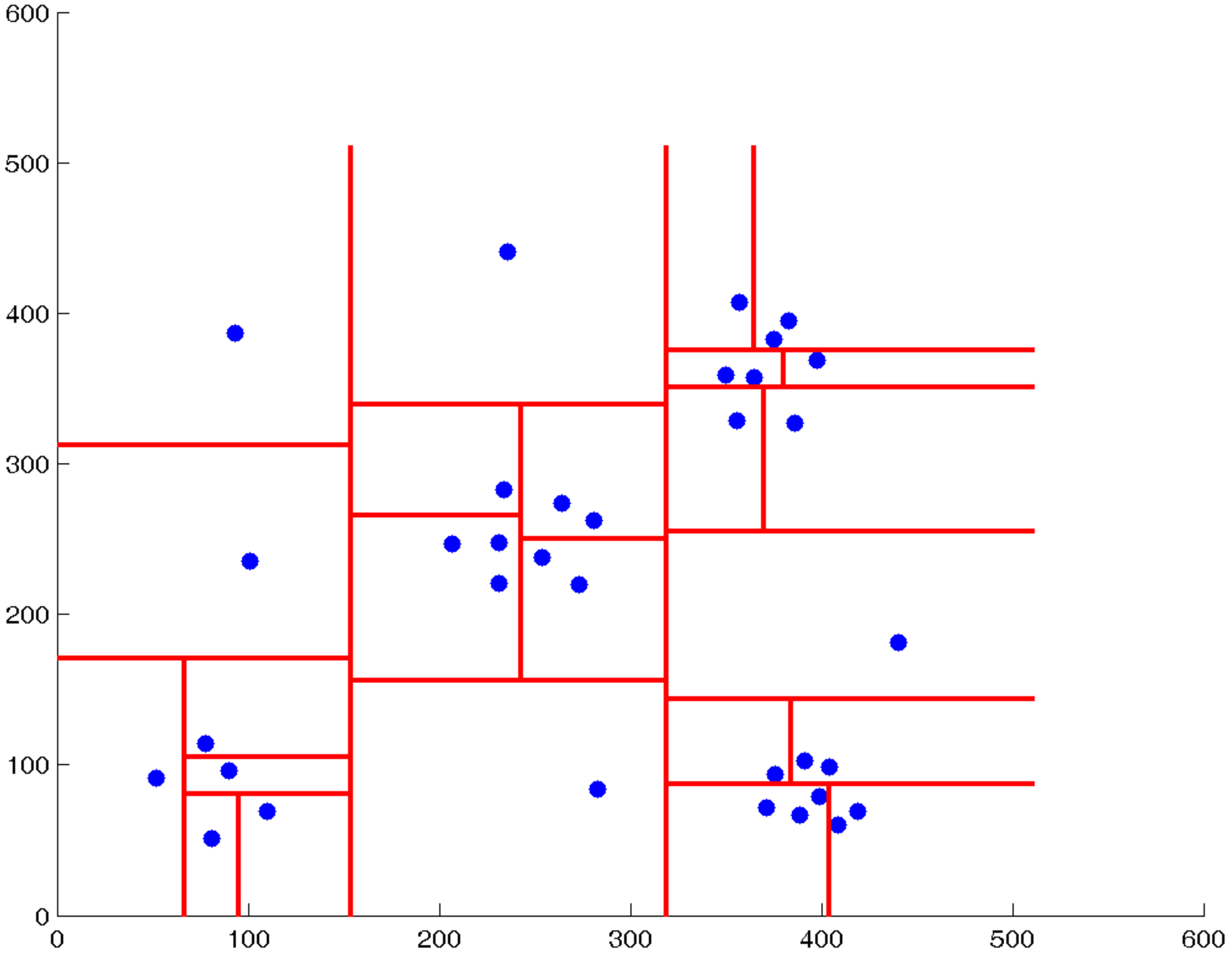
Layer 5



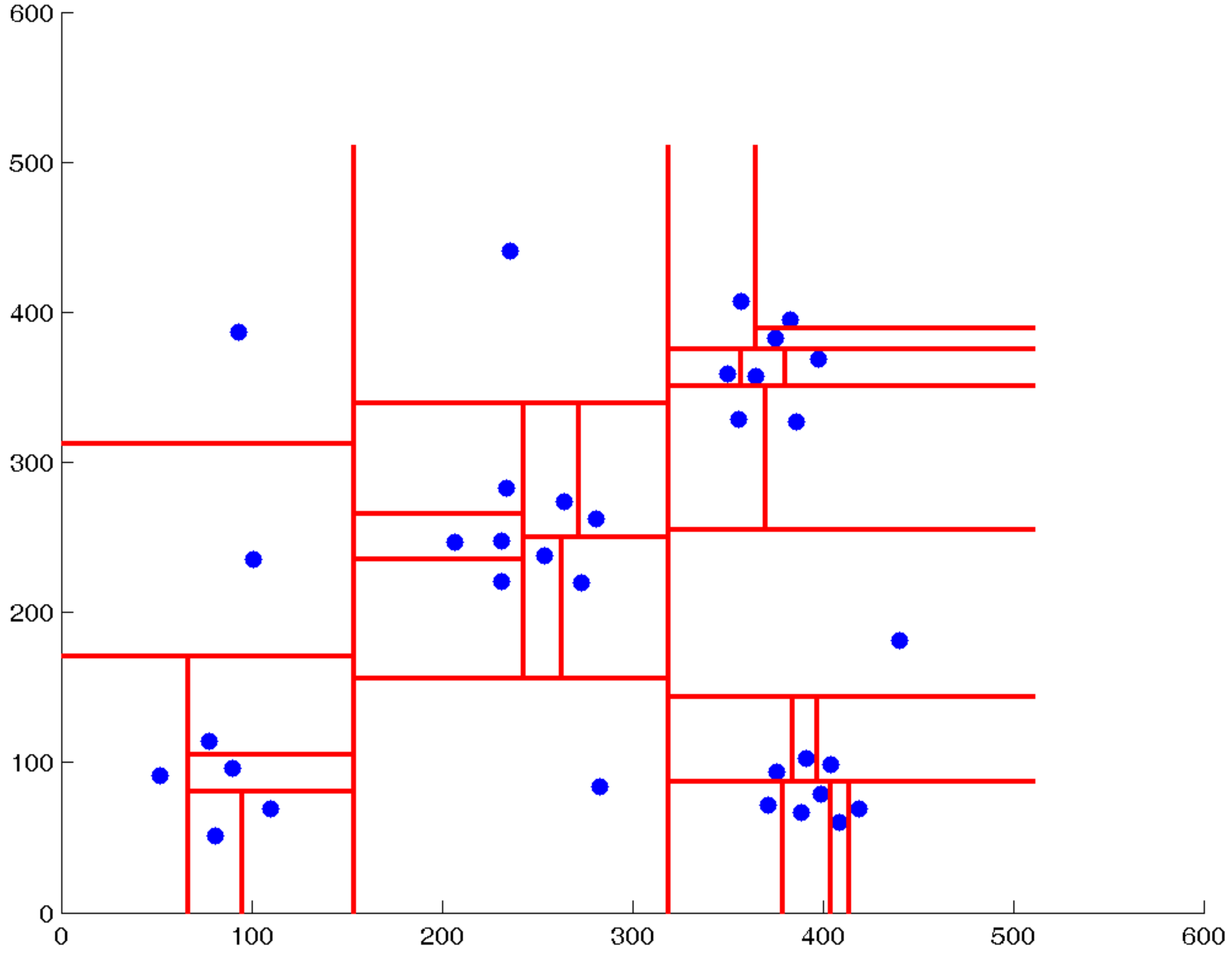
Layer 4



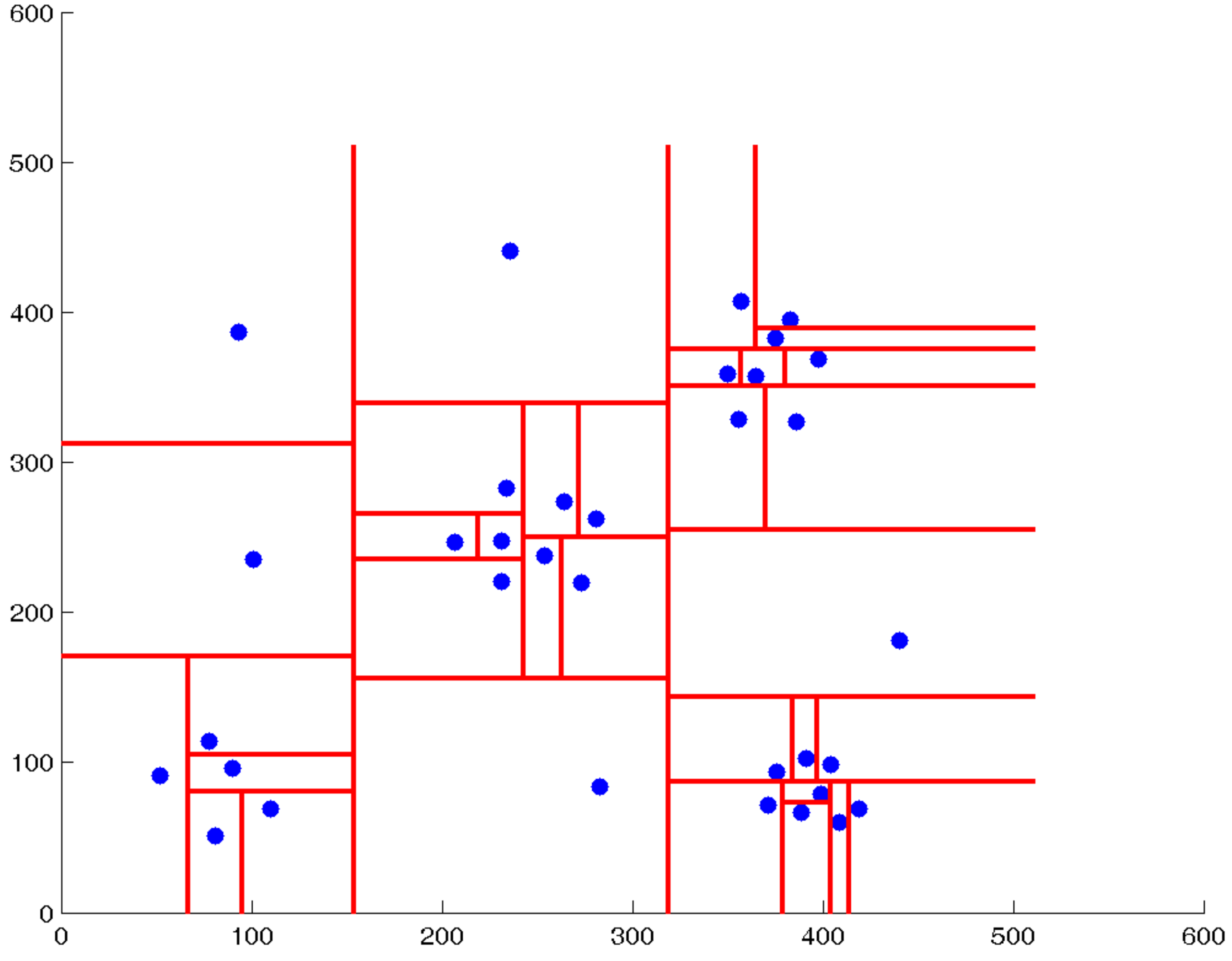
Layer 3



Layer 2



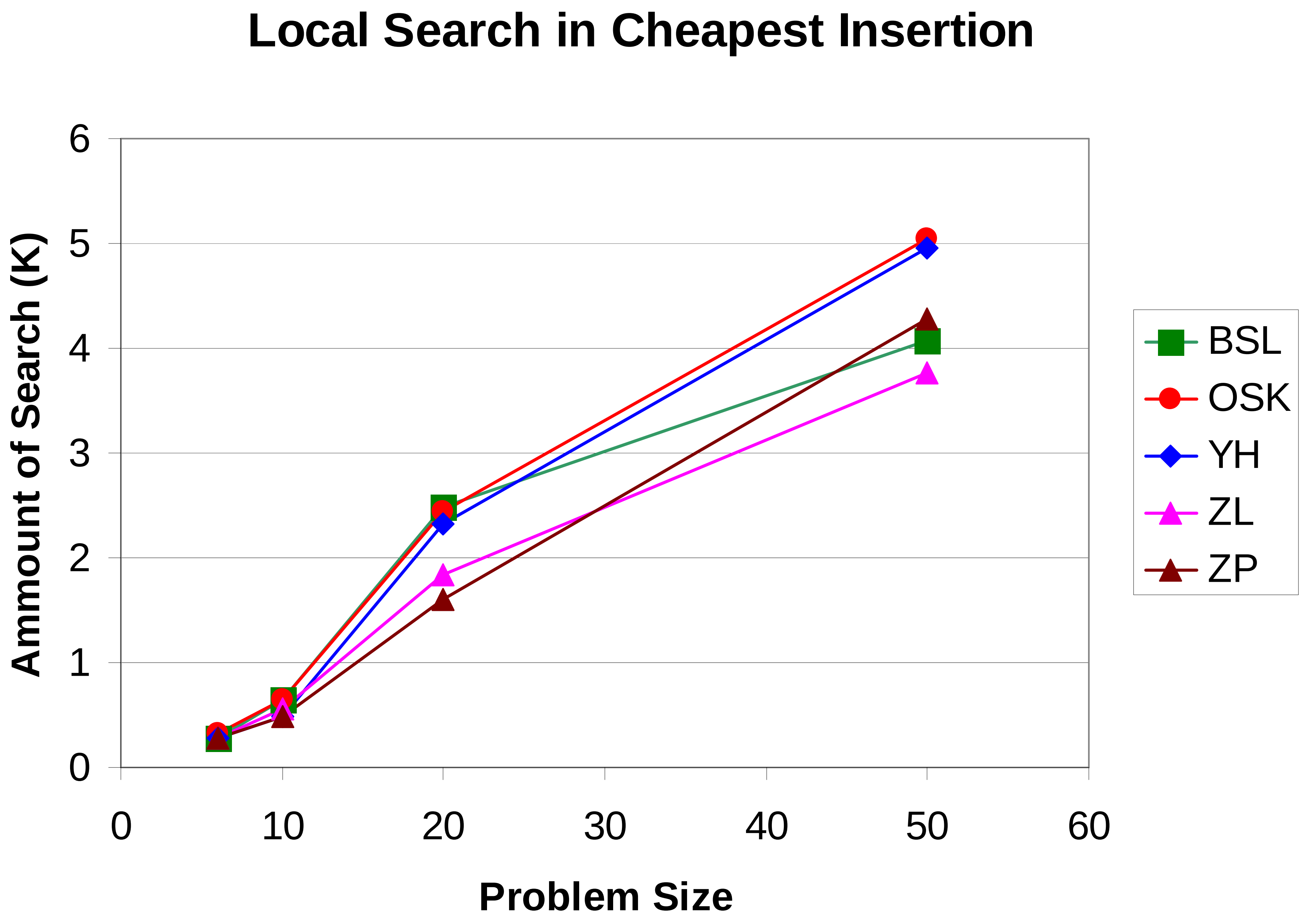
Layer 1



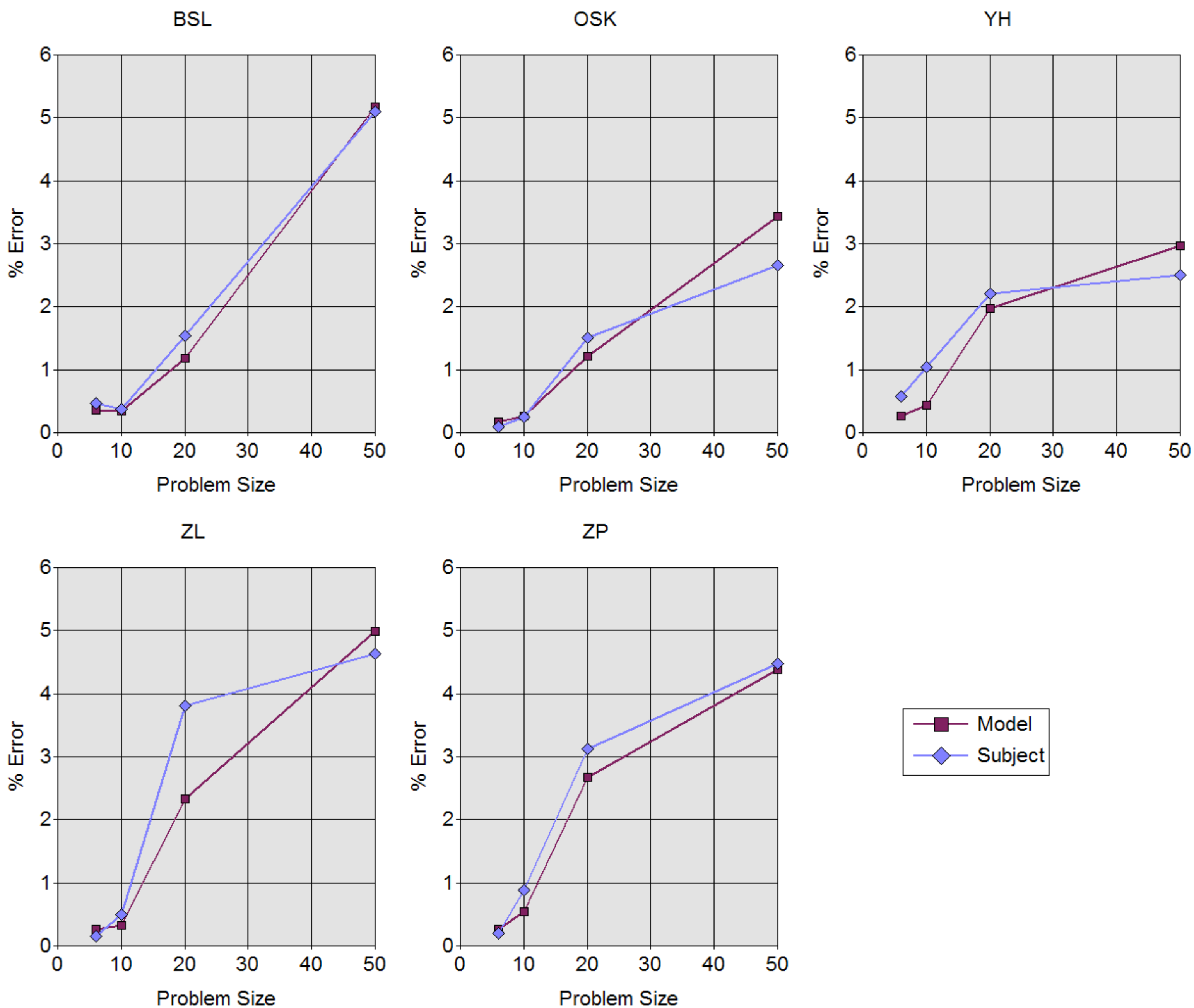
Testing the Pyramid Model

- The model was run on the same problems that were used with the subjects
- The size k of the neighborhood for cheapest insertion was a free parameter
- Computational complexity of the model: between $O(N)$ and $O(N^2)$.

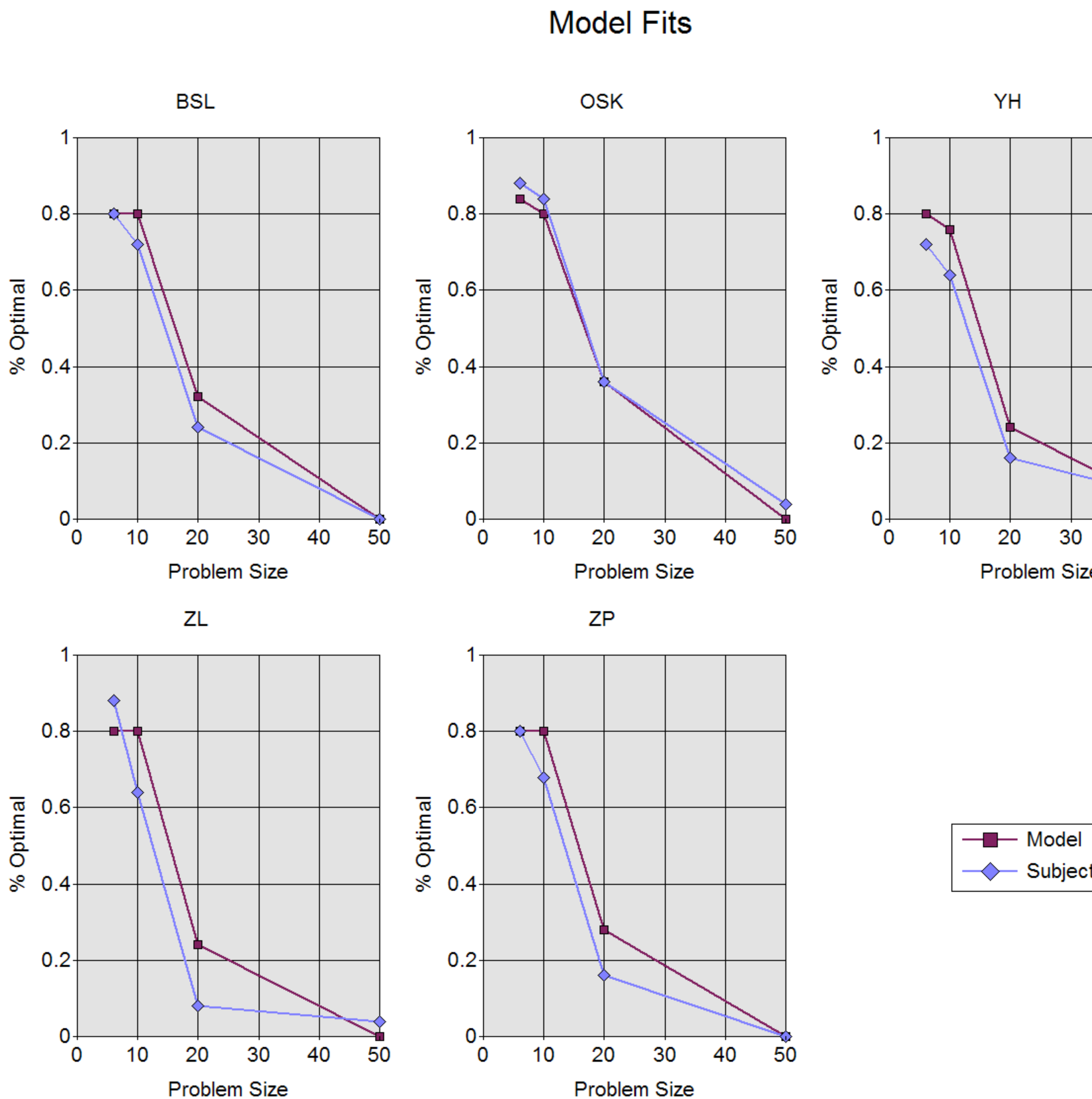
Demo


Local Search in Cheapest Insertion
Ammount of Search (K)
0
1
2
3
4
5
6
0
10
20
30
40
50
60
Problem Size
BSL
OSK
YH
ZL
ZP

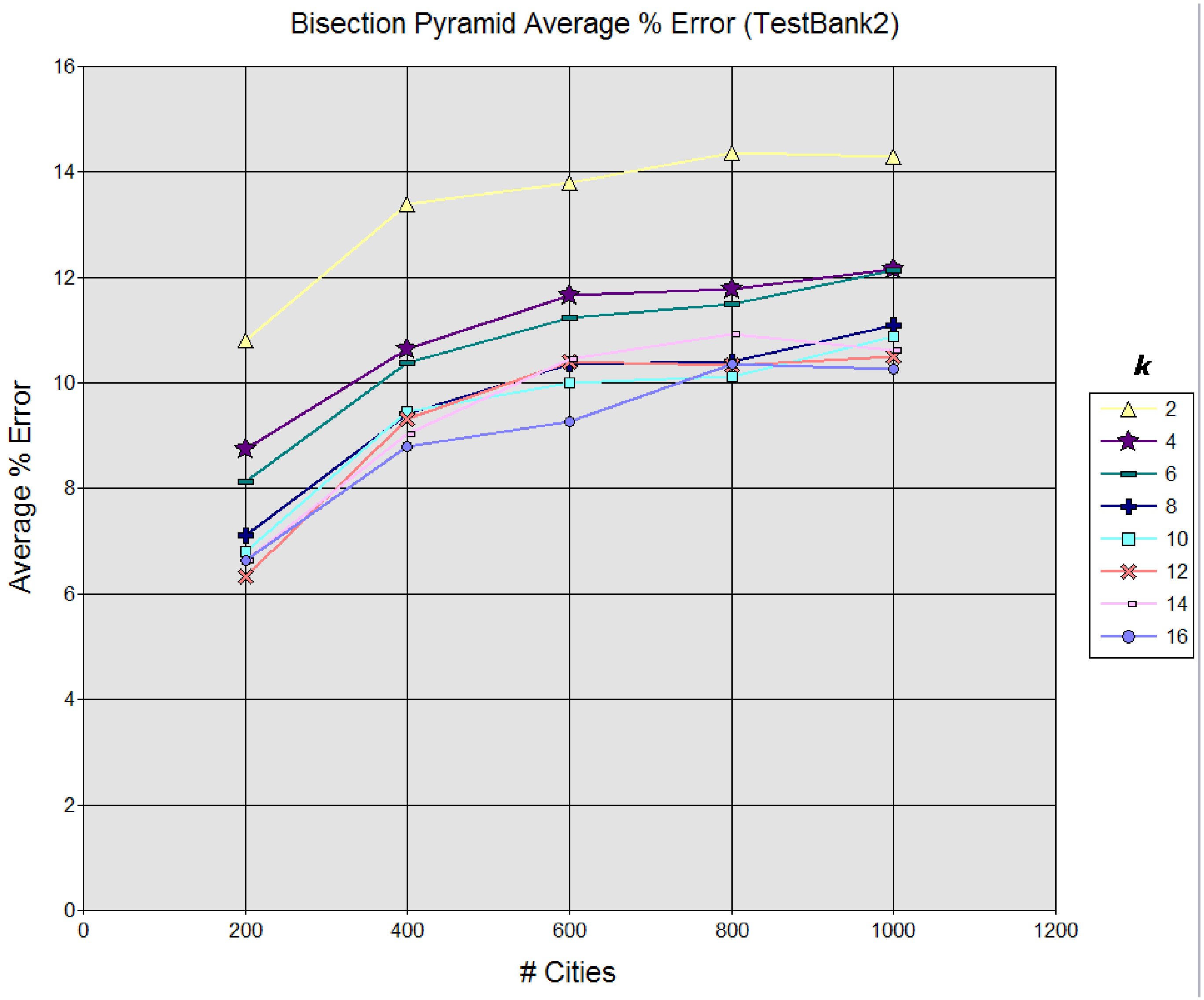
Model Fits



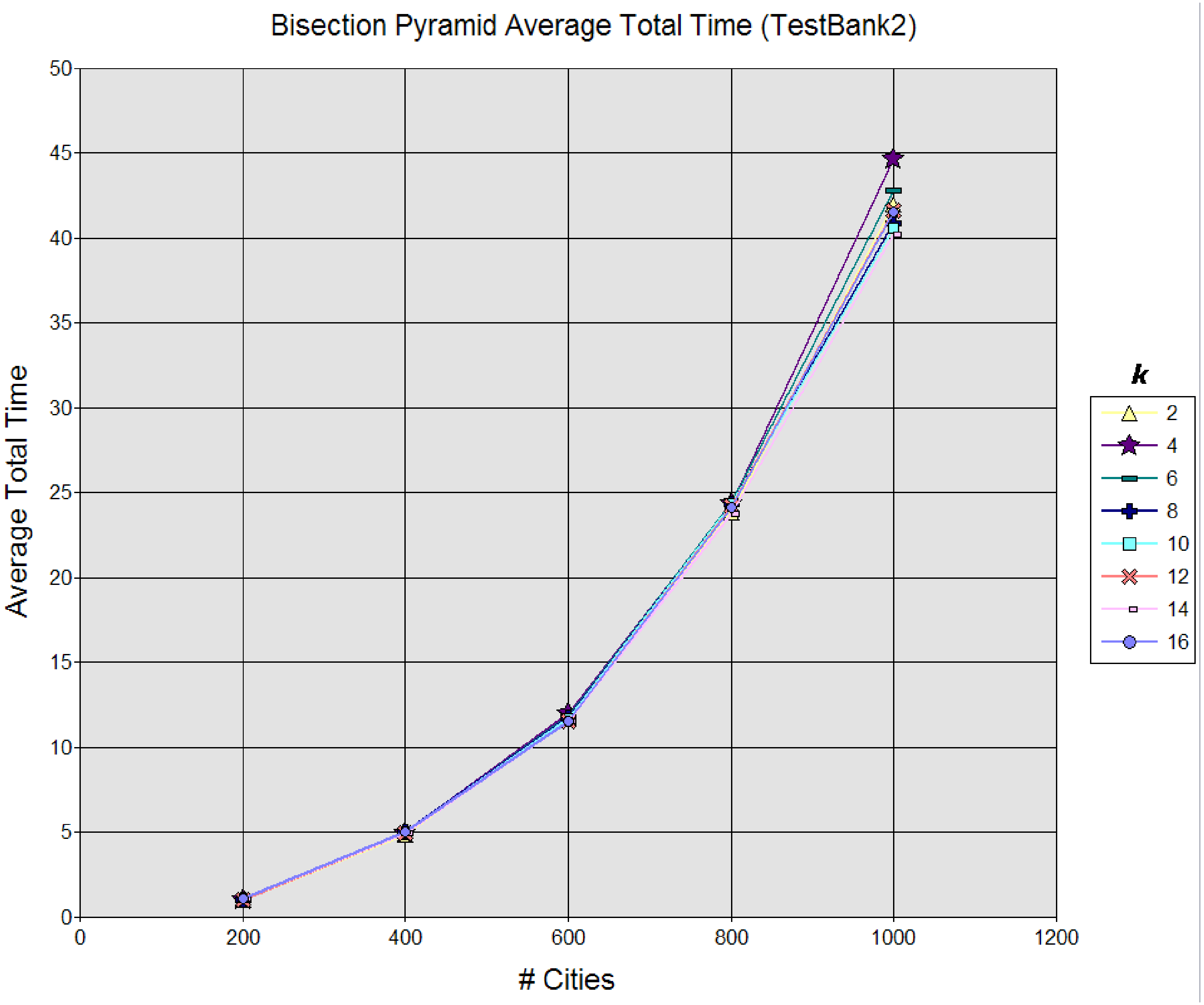
BSL
OSK
YH
ZL
ZP
% Error
Problem Size
0
1
2
3
4
5
6
10
20
30
40
50
Model
Subject


Model Fits
BSL
OSK
YH
ZL
ZP
% Optimal
Problem Size
0
0.2
0.4
0.6
0.8
1
0
10
20
30
40
50
Model
Subject

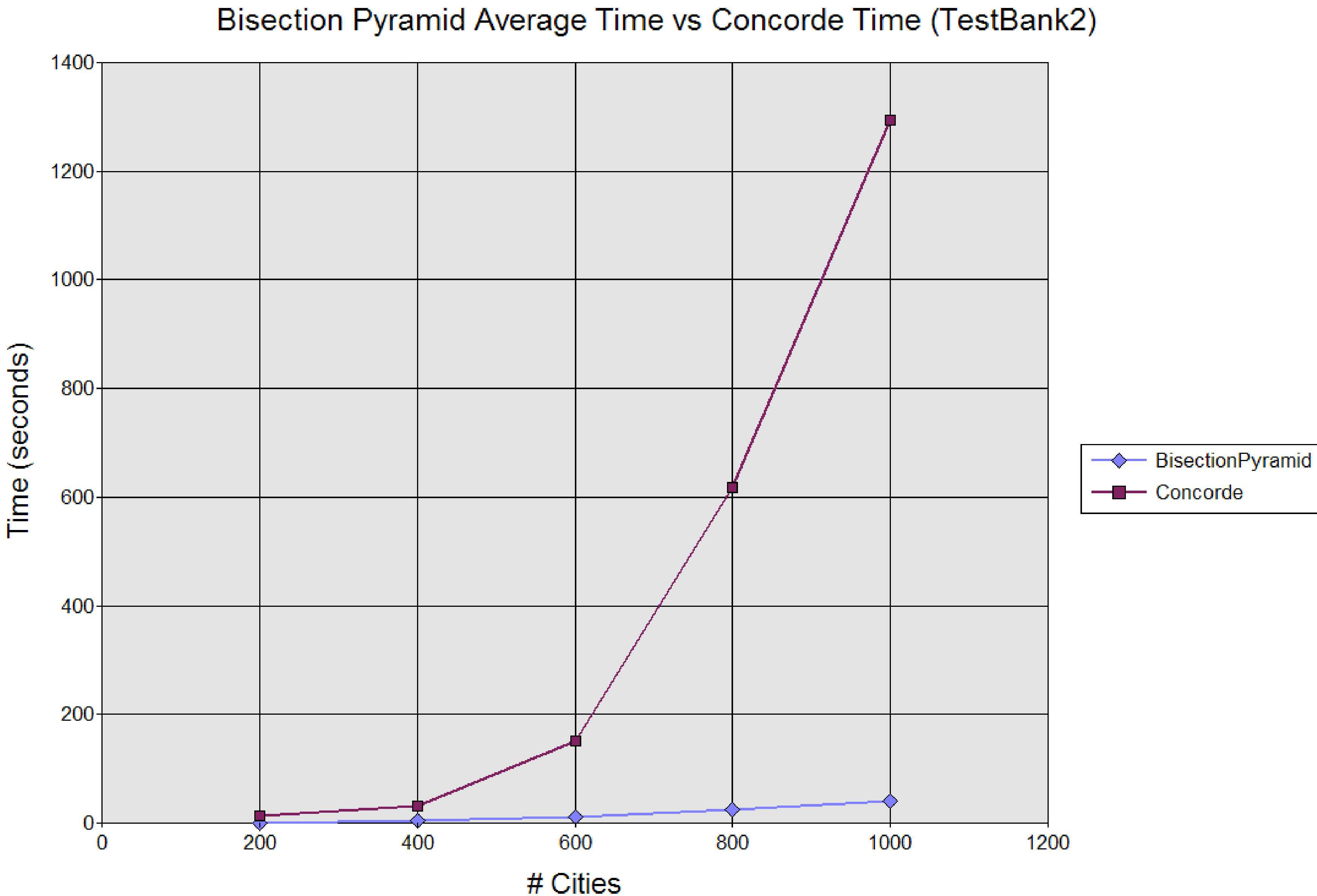
Large Problems



Large Problems



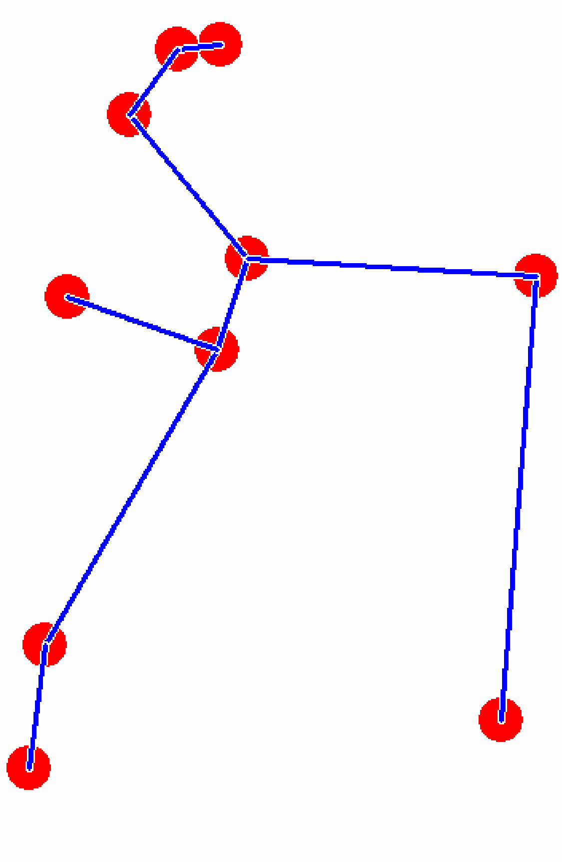
Large Problems



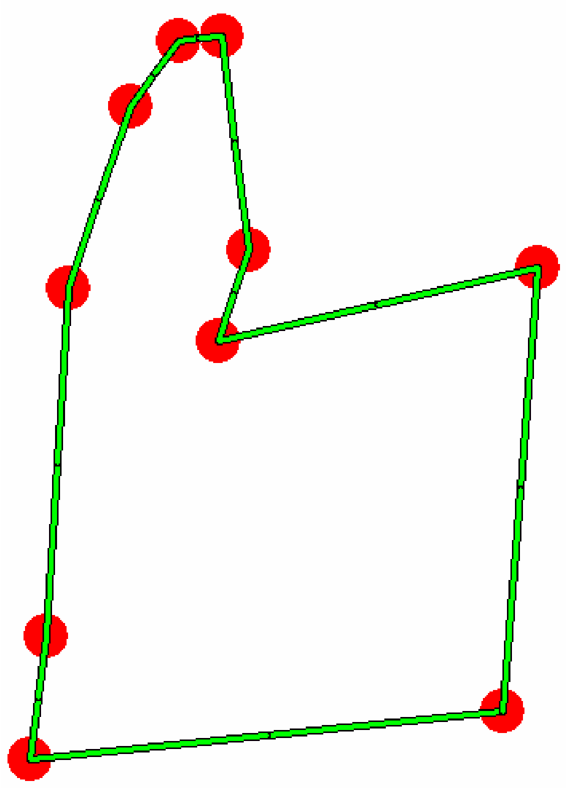
ZP solving large problems…

Minimum Spanning Tree vs. TSP

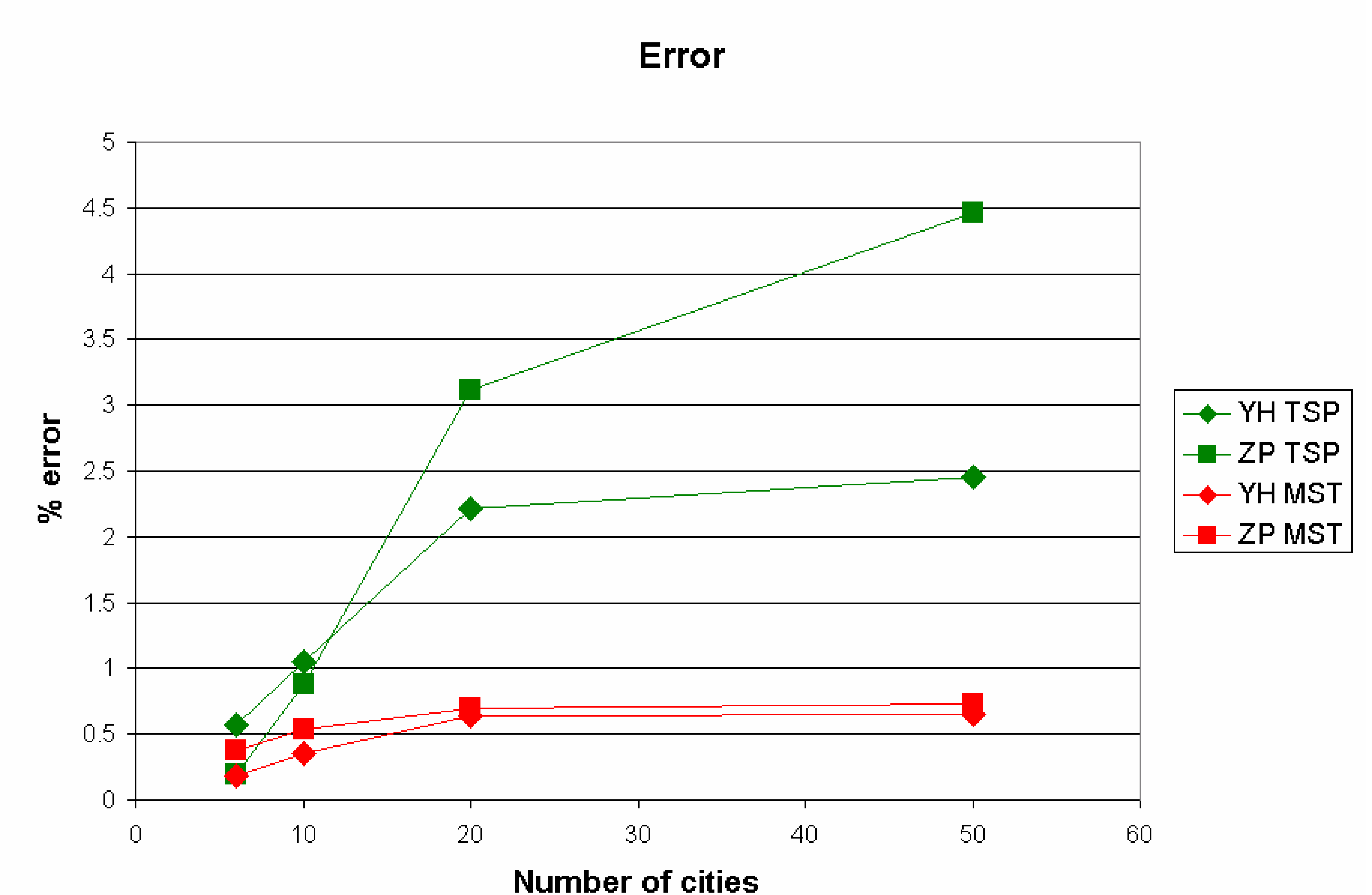
MST



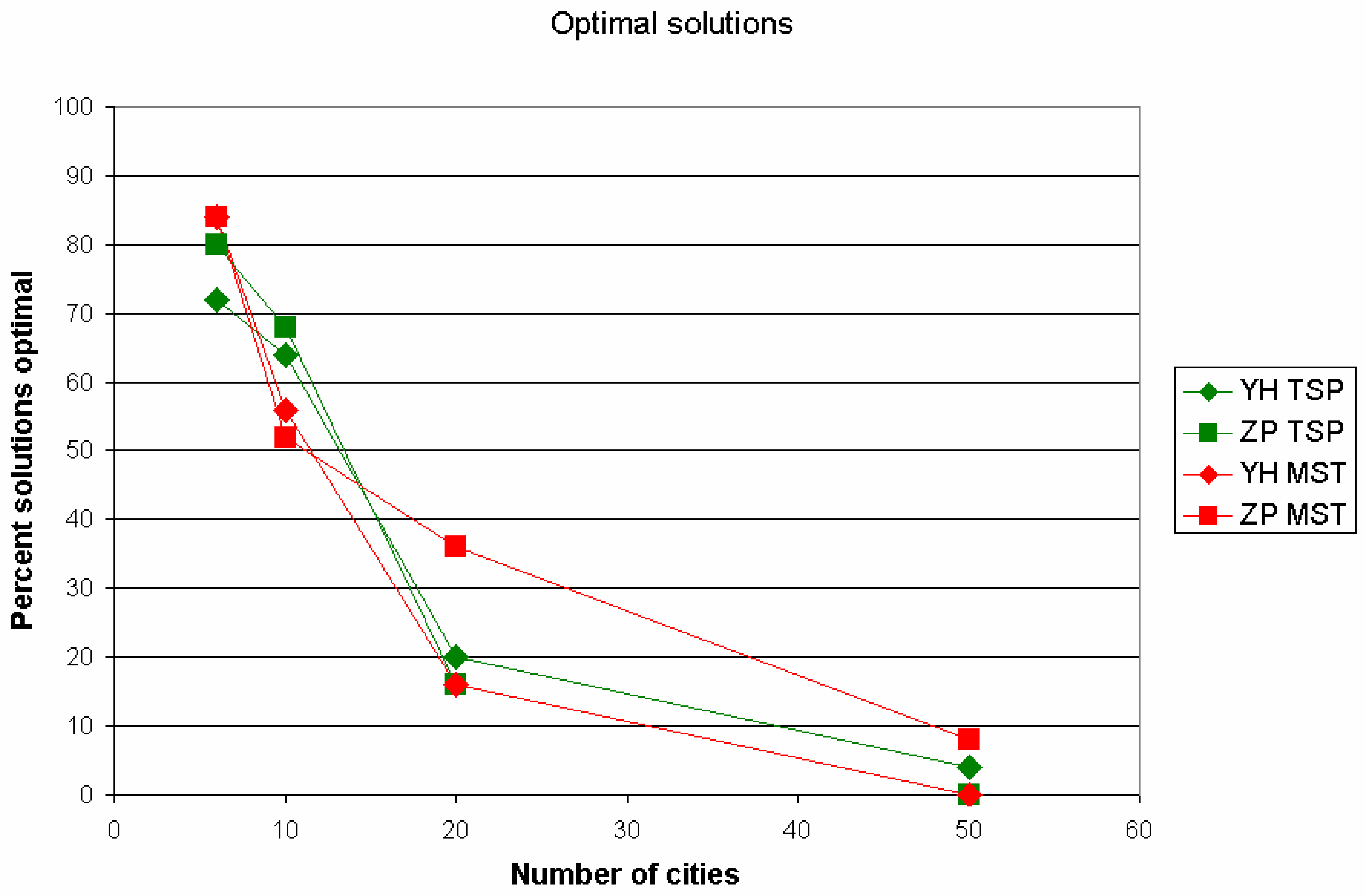
TSP



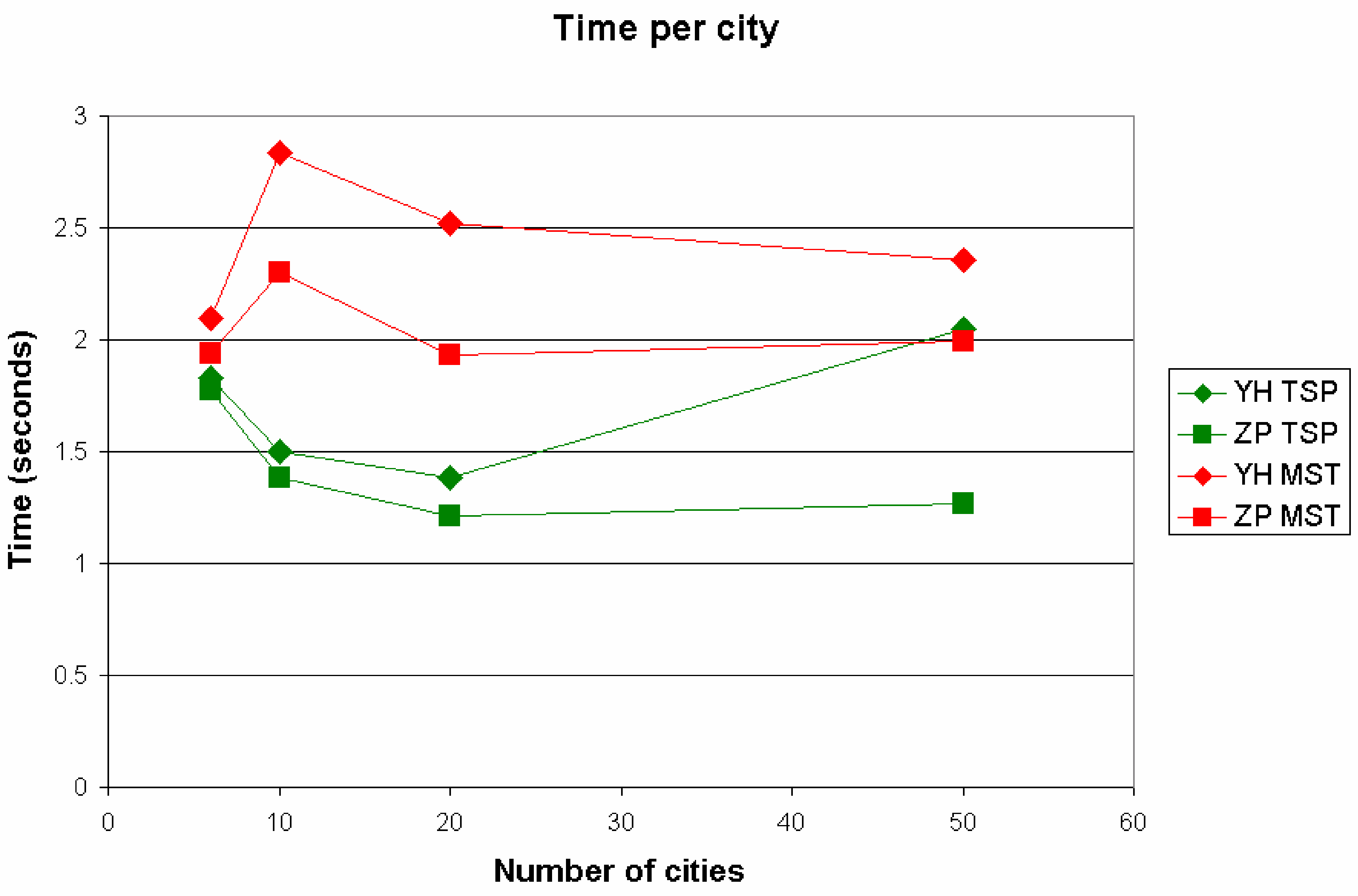
Psychophysics: MST vs. TSP



Psychophysics: MST vs. TSP



Psychophysics: MST vs. TSP

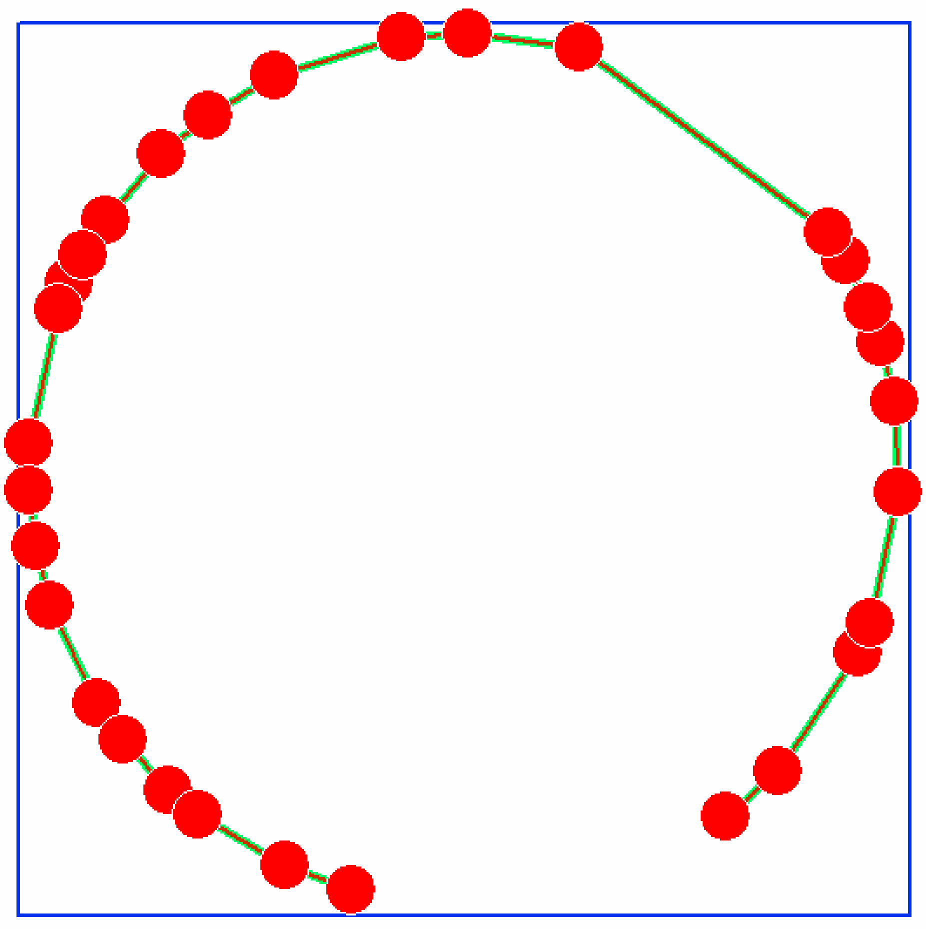


What is MST actually good for?

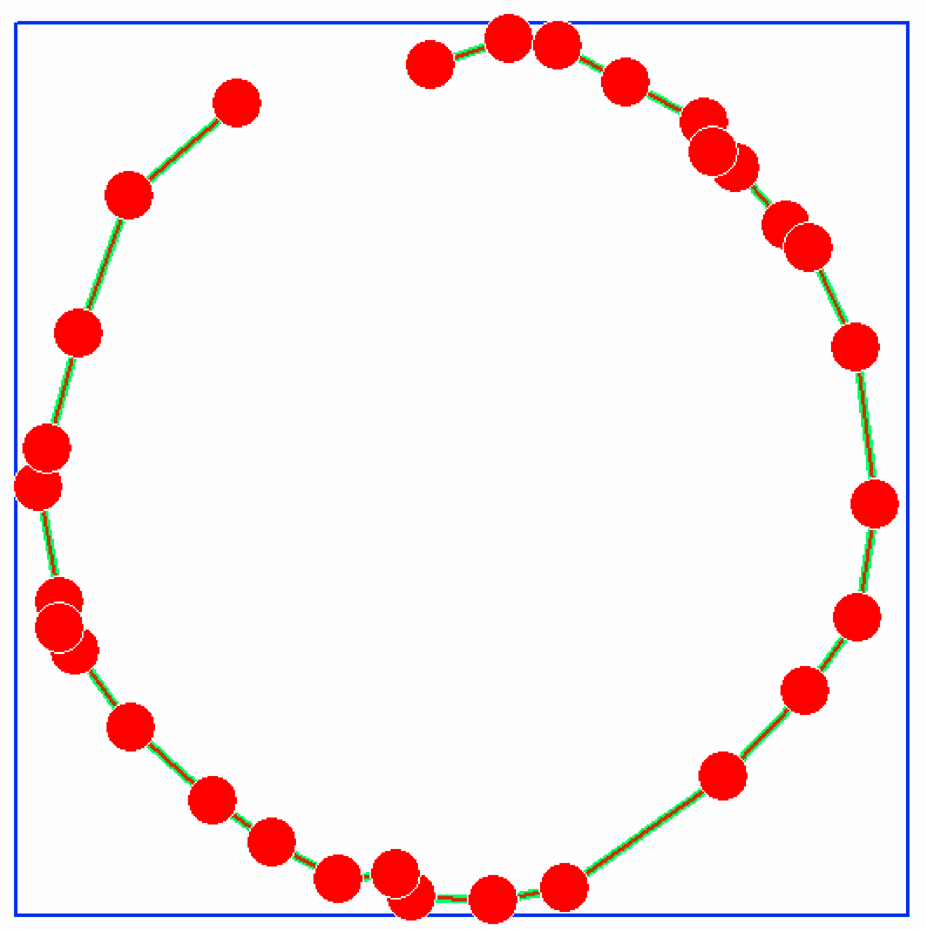
- Clustering?
- What type of clustering?

MST as line detector

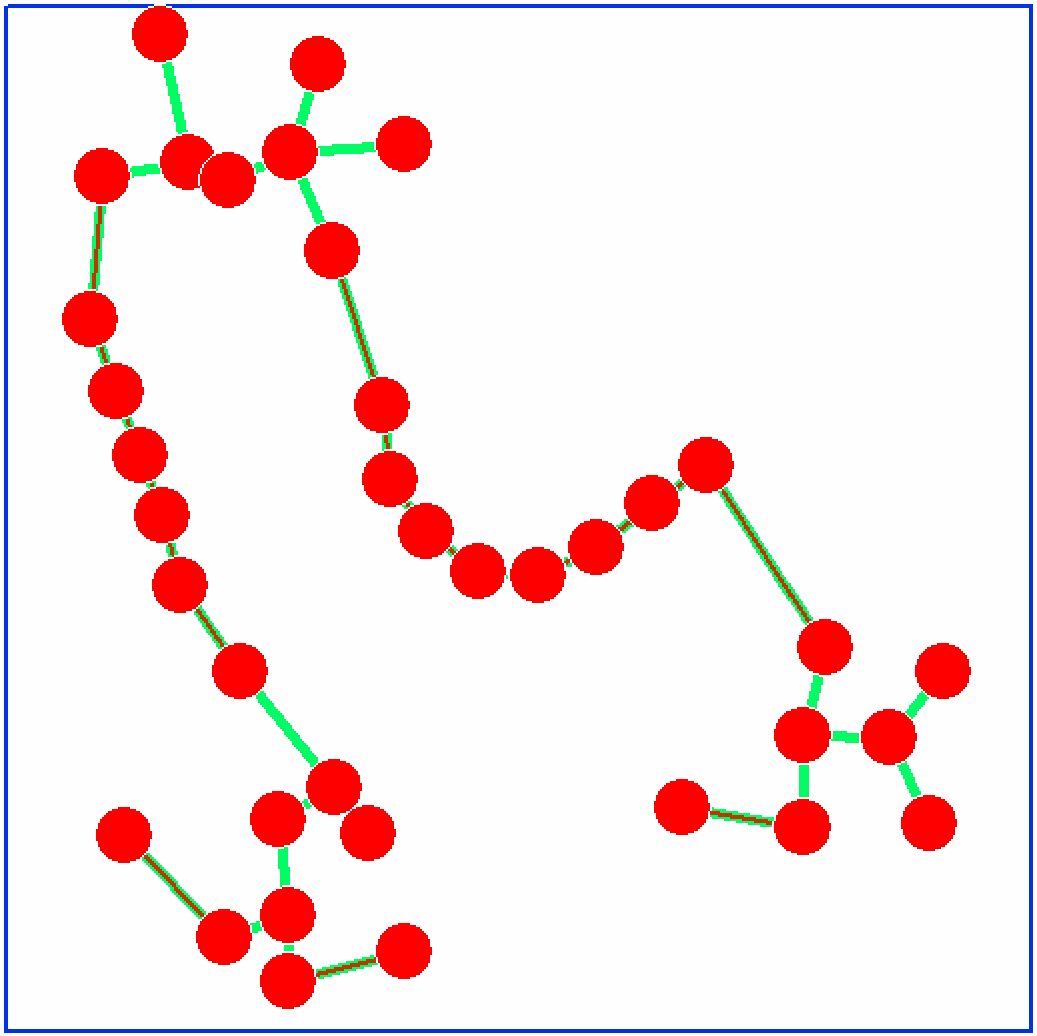
Perfect circle



Less-than perfect circle

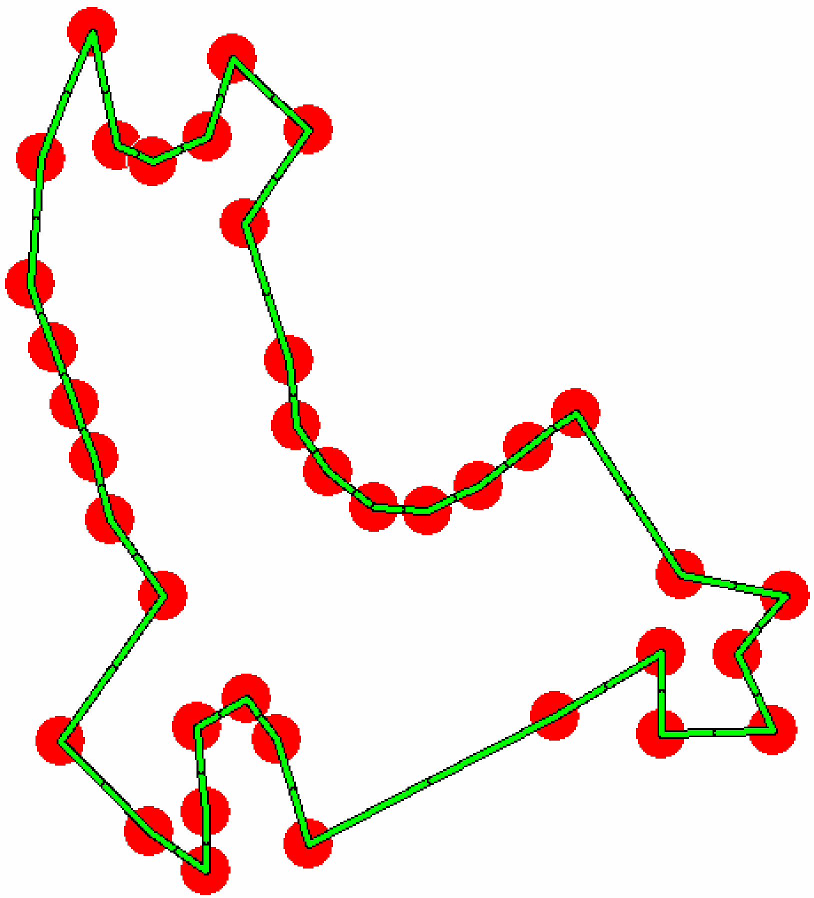


MST for a realistic example

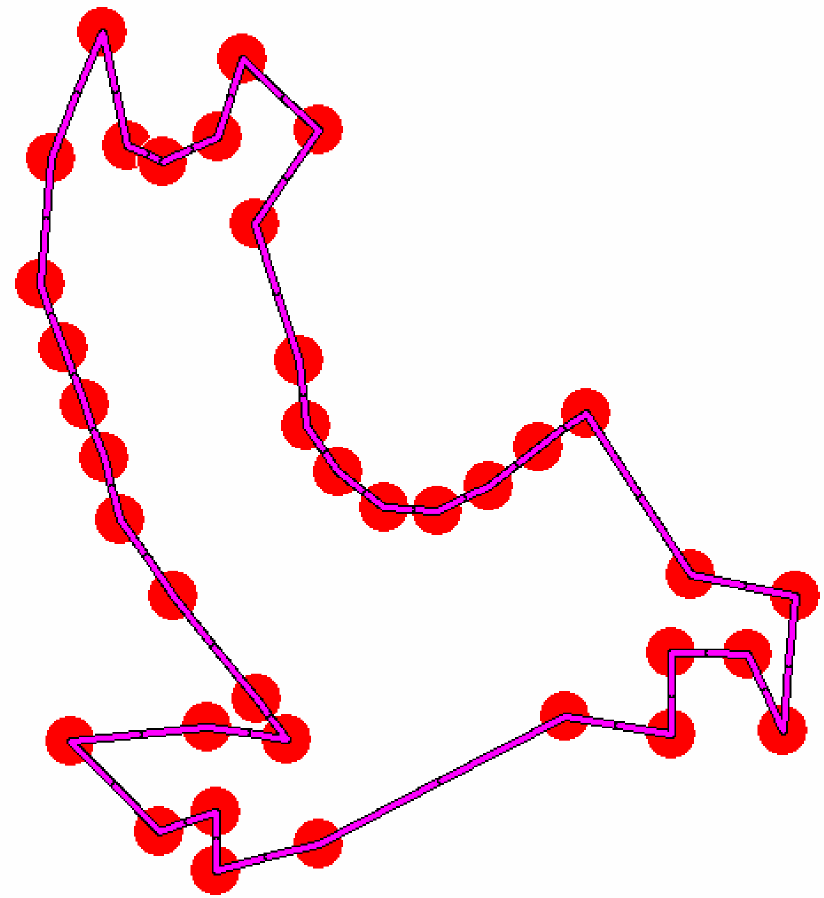


TSP solutions

Optimal



Line Pyramid



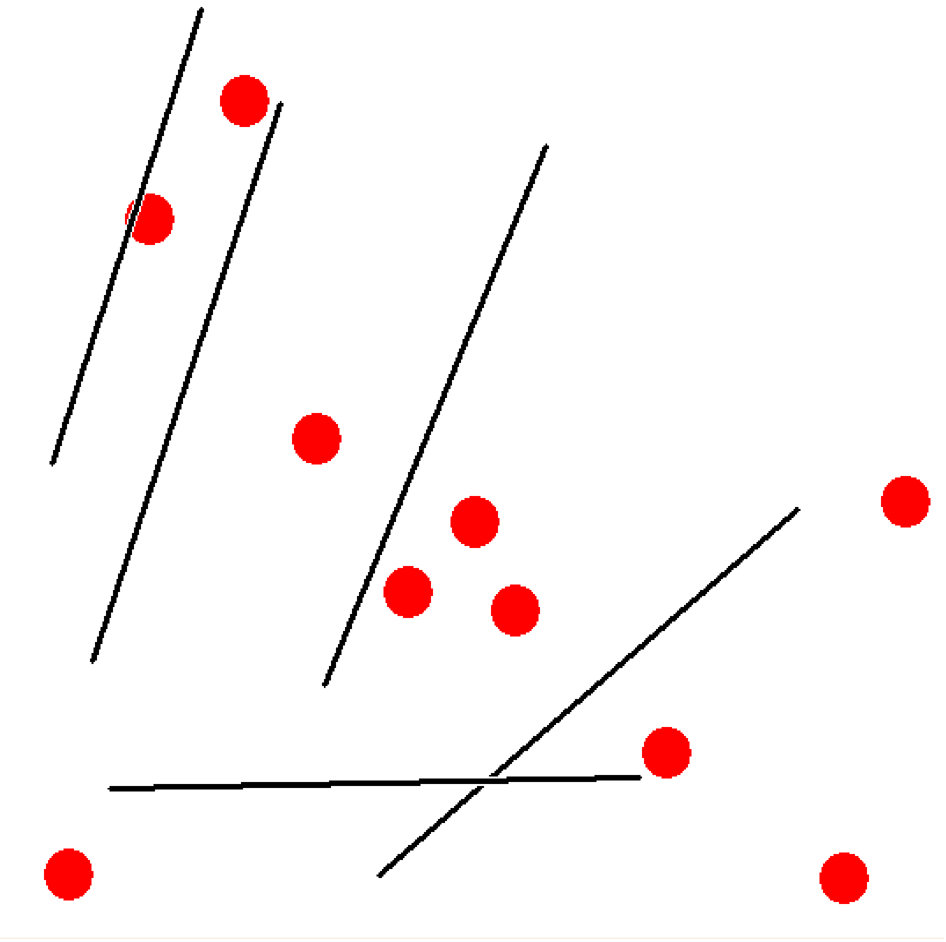
Summary

- Computational complexity of the mental mechanisms is very low but TSP tours found by the subjects are close to optimal.
- Coarse-to-fine sequence of approximations produced by a pyramid algorithm provides a plausible model of the mental mechanisms involved in solving TSP.
- The TSP model simulates attention (visual acuity), as well as eye movements – this minimizes the use of memory without slowing down the solution process.
- Simulated receptive fields are adaptive.
- The line detection mechanism is likely to be based on MST.

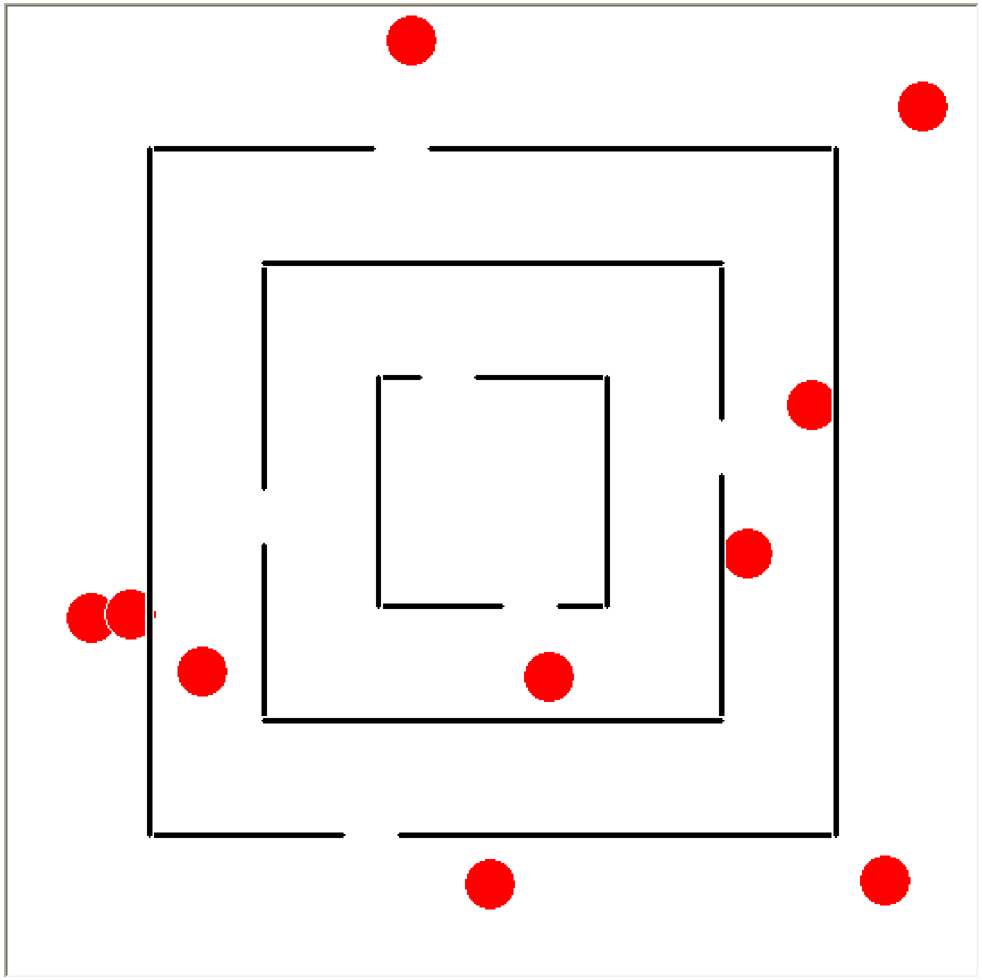
Next Step

- Test the model using TSP with obstacles.

Euclidean TSP with Obstacles
(NE-TSP)

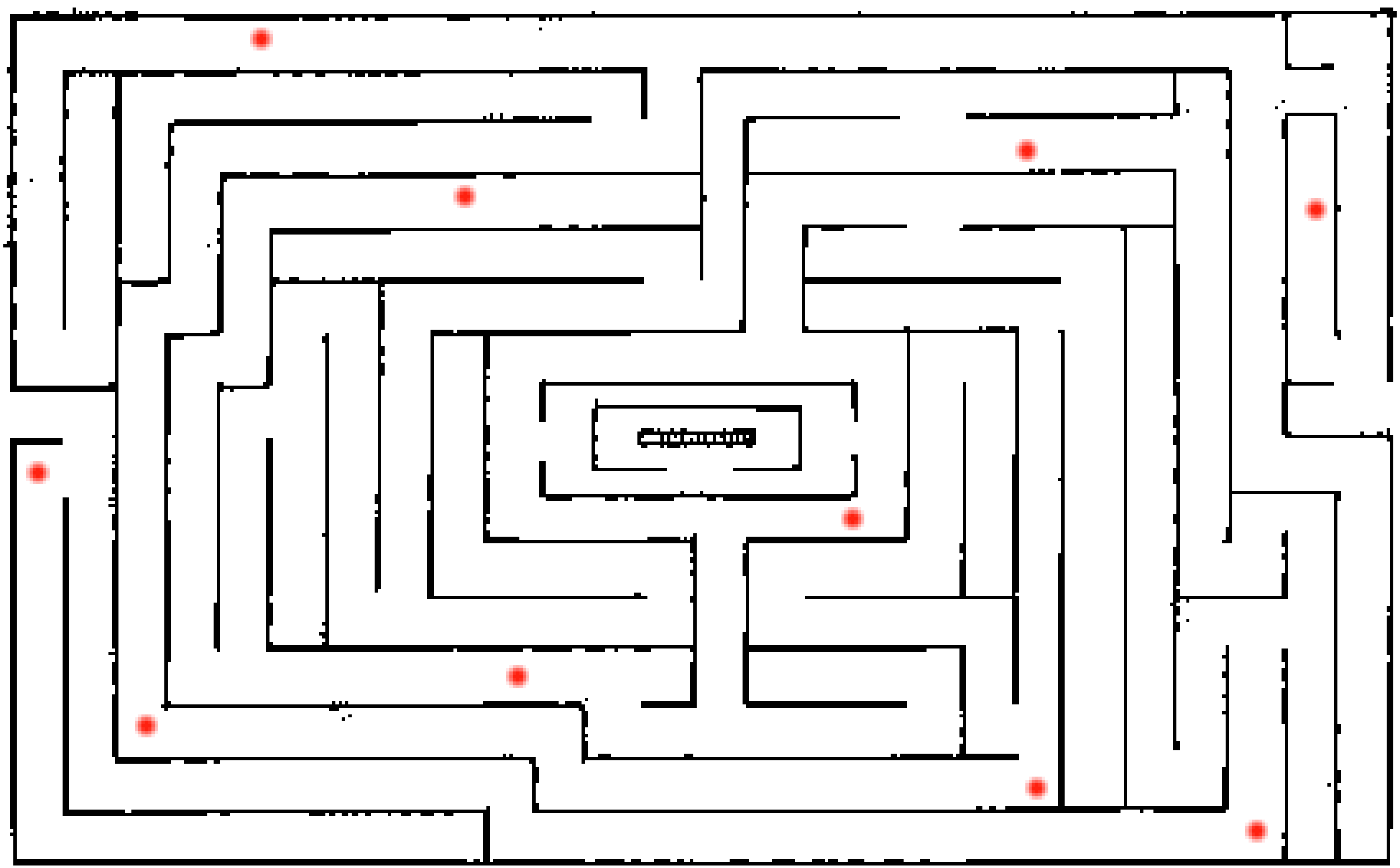


Maze – Like Obstacles



Visual spatial relations in the problem representation (proximities, directions) have to be modified by bottom-up verification of availability of moves.

Metric Always Exists, but May be Difficult to Reconstruct



Maze at Hatfield House, Herts