PSY 201: Statistics in Psychology

Lecture 18

Hypothesis testing of the mean

Why I don't use herbal medicines.

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SUPPOSE

- we think the mean value of a population of SAT scores is $\mu = 455$
- we can take a sample of the population and calculate the sample mean of SAT scores $\overline{X} = 535$
- we can make some statement about how rare it is to get a result like $\overline{X} = 535$ (what we did last time)
- **and** if such a result is very rare
- we can make a statement about how unreasonable it is that our original thought is true!

HYPOTHESIS TESTING

- in hypothesis testing we consider how reasonable a hypothesis is, given the data that we have
- if the hypothesis is reasonable (consistent with the data), we assume it could be true
- if the hypothesis is unreasonable (inconsistent with the data), we assume it is false
- deciding on what hypotheses to test is critically important!

HYPOTHESIS TESTING

- four steps:
  1. State the hypothesis and criterion.
  2. Compute the test statistic.
  3. Compute the p value.
  4. Make a decision.

HYPOTHESIS

- conjecture about one or more population parameters
- e.g.
  - $\mu = 455$
  - $\mu_1 = \mu_2$
  - $\sigma = 3.5$
  - $r = 0.76$
  - ...
- in inferential statistics we always test the **null hypothesis**: H_0

NULL HYPOTHESIS

- H_0 is the assumption of no relationship, or no difference. e.g.
  - H_0: no relationship between variables
  - H_0: no difference between treatment groups
- We want the H_0 to be *specific* so that we can define a sampling distribution
- the alternative hypothesis, H_a is the other possibility. e.g.
  - H_0: $\mu = 455$
  - H_a: $\mu \neq 455$
- does not say what μ is, but says what it is not!

NULL HYPOTHESIS

- what's wrong with herbal medicines?
- nothing necessarily, but I don't know that they are any good (and they may be bad)
- lots of reports that they help people (but how can they be sure)
- need to start by assuming that a medicine does nothing, and **prove** that the assumption is false!
- anecdotal reports are just about worthless

NULL HYPOTHESIS

- often times (almost always) the goal of statistical research is to reject the null hypothesis, so that the only alternative is to accept H_a
- similar to an indirect proof. e.g.
  - show that the angles of a triangle sum to 180^o by assuming that they do not and then finding a contradiction
- why this approach?
  - it is much easier to show that something is false (H_0) than to show that something is true (H_a)
- understanding of relationship between variables or differences between groups often requires many experiments!

STATE THE HYPOTHESIS

- before doing anything else, we need to make certain that we understand the tested hypothesis
- for the SAT example

$$H_0 : \mu = 455$$

$$H_a : \mu \neq 455$$

- sometimes this is the most difficult step in designing an experiment
- to start, we will worry only about hypotheses about the population mean, μ

SIGNAL DETECTION

- The task is almost the same as deciding whether a measurement came from a noise-alone (null hypothesis) distribution or a signal-and-noise (alternative hypothesis) distribution
- How well you can do is determined by the signal-to-nose ratio (d'), but that value is typically unknown
- we set a criterion using only the null hypothesis (noise-alone distribution)

CRITERION

- we will examine the data to see if we should reject H_0
- we will do that by comparing the sample mean, $\overline{X}$, to the hypothesized value of the population mean, μ
- the bottom-line is whether $\overline{X}$ is sufficiently different from μ to reject H_0
- but we have to consider four things to quantify the term *sufficiently different*
  - standard scores
  - errors in hypothesis testing
  - level of significance
  - region of rejection

STANDARD SCORES

- we previously used standard scores to indicate how much a given score deviates from a distribution mean
- We do the same kind of thing here, but we want to know how a sample mean, $\overline{X}$ deviates from what the sampling distribution would be if the null hypothesis is true
- We give the standard score a special term:

$$t = \frac{\overline{X} - \mu}{s_{\overline{X}}}$$

- We compute everything else using the sampling distribution of this t value: the t distribution, which is similar to a normal distribution with fatter tails and requires degrees of freedom:

$$df = n - 1$$

DECISIONS

- after deciding to reject or not reject H_0 there are four possible situations
  - A true null hypothesis is rejected. (False alarm)
  - ** A true null hypothesis is not rejected. (Correct rejection)
  - A false null hypothesis is not rejected. (Miss)
  - ** A false null hypothesis is rejected. (Hit)
- errors are unavoidable
- we want to minimize the probability of making errors, given the particular data set we have

ERRORS

- two types of errors:
  - **Type I error**: when we reject a true null hypothesis (false alarm).
  - **Type II error**: when we do not reject a false null hypothesis (miss).

	State of nature	
Decision made	H_0 true	H_0 false
Reject H_0	Type I error	Correct decision
Do not reject H_0	Correct decision	Type II error

- generally, decreasing the probability of making one type of error increases the probability of making the other type of error

ERRORS

- suppose you have a new, untested, and expensive treatment for cancer
- you run a test to judge whether the drug is better than existing drugs
- if you reject H_0, indicating that the drug **is** more effective, when in fact it is not, people will spend a lot of money for no reason (Type I error)
- if you fail to reject H_0, indicating that the drug is not effective, when in fact it is, people will not use the drug (Type II error)
- scientific research tends to focus on avoiding Type I errors

SIGNIFICANCE LEVEL

- alpha (α) level
- indicates probability of Type I error
- frequently we choose $\alpha = 0.05$ or $\alpha = 0.01$
- that is, the corresponding decision to reject H_0 may produce a Type I error 5% or 1% of the time
- a statement about how much error we will accept
- usually chosen **before** the data is gathered
  depends upon use of the analysis

REGION OF REJECTION

- α is a probability
- it identifies how much risk of Type I error we are willing to take (rejecting H_0 when it is true)
- consider our example of SAT scores

$$H_0 : \mu = 455$$

- suppose we also know the sample standard deviation

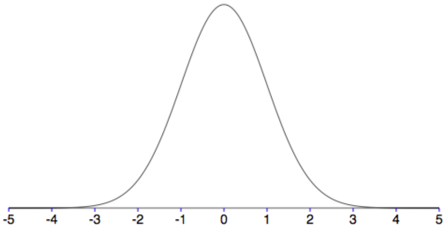
$$s = 100$$

- and our sample size is $n = 144$

REGION OF REJECTION

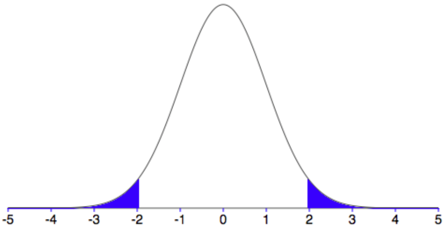
- we know that the sampling distribution of t is:
  - A t distribution with $df = n - 1 = 143$.
  - Has a mean of $\mu = 0$, if H_0 is true
  - Has a standard error of the mean

$$s_{\overline{X}} = \frac{s}{\sqrt{n}} = \frac{100}{\sqrt{144}} = 8.33$$



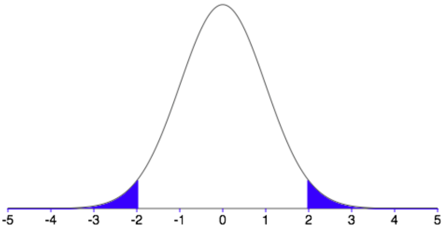
REGION OF REJECTION

- area under the curve represents the probability of getting the corresponding t values, if the H_0 is true
- the extreme tails of the sampling distribution correspond to what should be very rare t values, and thus very rare sample means



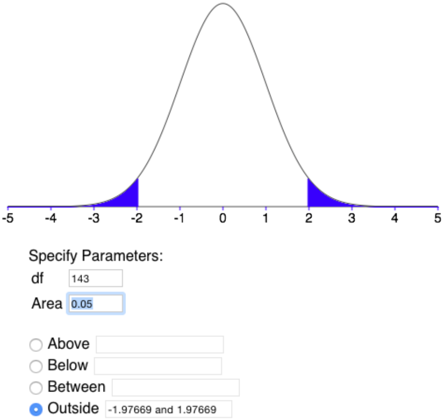
REGION OF REJECTION

- we shade in the extreme α percentage of the sampling distribution
- called the region of rejection
- if our data produces a t value in the region of rejection, we reject H_0 because it is unlikely that we would get such a value if the H_0 were true.



REGION OF REJECTION

- values of sample means at the beginning of the region of rejection
- NOTE: α is split up in each tail
- called a two-tailed or non-directional test



TEST STATISTIC

- if the t-score is beyond ± 1.977, it is very unlikely to have occurred if the H_0 is true.
- we have the following data:
  - $\mu = 455$, H_0
  - $n = 144$, sample size
  - $\overline{X} = 535$, observed value for sample statistic
  - $s = 100$, value of the standard deviation of the population
  - $s_{\overline{X}} = 8.33$, standard error (calculated earlier)
- from this we can calculate the t-score

TEST STATISTIC

- we want to know how different $\overline{X}$ is from the hypothesized μ in terms of standard error units

$$t = \frac{\overline{X} - \mu}{s_{\overline{X}}}$$

$$t = \frac{535 - 455}{8.33} = 9.60$$

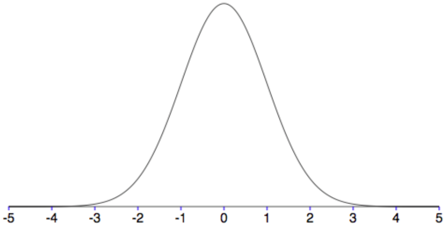
- the standard score is the **test statistic** for testing H_0 about a population mean

DECIDING ABOUT H_0

- compare the test statistic to the critical value

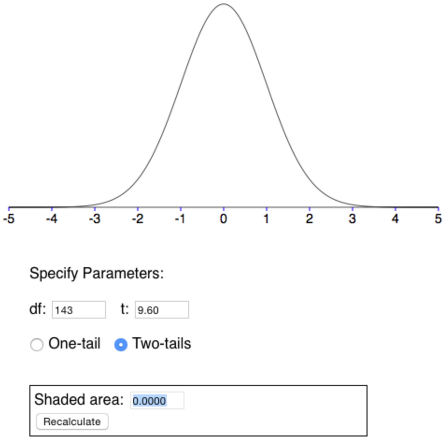
$$t = 9.60 > 1.977 = t_{cv}$$

- indicates that the sample mean $\overline{X}$ is extremely rare, given the assumed population mean μ, by chance (random sampling)



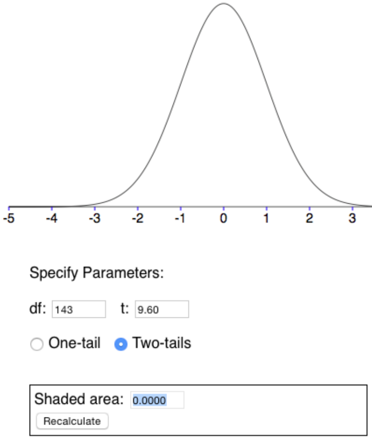
p-VALUE

- another way to do it (advocated by your text) is to use the *t*-value to compute the probability of getting a *t*-value more extreme than what you found
- *p*-value
- *t* distribution calculator



p-VALUE

- We find $p \approx 0$
- Since the probability is small ($< .05$), then we conclude that the H_0 is probably not true



DECISIONS

- since the p value is smaller than the α we set, we reject

$$H_0 : \mu = 455$$

- in favor of the alternative hypothesis

$$H_a : \mu \neq 455$$

- but there is still a chance that H_0 is true!

CONCLUSIONS

- null hypothesis
- rejecting H_0
- Type I error
- Type II error

NEXT TIME

- Test statistic
- Deciding about H_0

Why clinical studies use thousands of subjects.